MULTIRESOLUTION FINITE ELEMENT CONTRAST SOURCE INVERSION METHOD

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Electromagnetic inverse scattering problems are relevant to a broad range of applications, including biomedical imaging, subsurface prospecting and security monitoring, only to mention some examples. For such a reason, there is always a huge interest in developing effective inversion methods capable to provide reliable imaging results.

In this respect, the ill-posedness and the non-linearity of the inverse scattering problem are the main difficulties to be faced, since, especially in presence of noise on data, they may cause the occurrence of inaccurate or even "false" solutions. To overcome these problems, it is necessary to devise suitable regularization strategies capable to drive the inversion process towards a correct (even if approximated) estimate of the unknown function. Among the various strategies exploited in the literature, one that is particularly effective is the so-called *regularization-by-projection*. In this method, the unknown of the problem, i.e., the contrast function that embeds the variation of the electric properties of the unknown target with respect to a known scenario, is represented in terms of suitable basis functions (e.g., the Fourier basis), so that the inversion is cast in terms of the representation coefficients (e.g., the Fourier coefficients). This strategy allows to significantly reduce the number of unknown parameters, which is a crucial outcome, since the reliability of the inversion process increases with the ratio among the available data (which belong to an essentially finite dimensional space) and the number of unknown parameters [*T. Isernia et al., IEEE Trans. Geosci. Remote Sens., 39, 1596-1607, 2001*].

Wavelet basis functions provide a powerful tool to implement regularization by projection, owing to their intrinsic capability of encoding a function in terms of coarse and fine details, which in turn allows to exploit a reduced set of coefficients to achieve a reliable approximation. For instance, wavelets can be used to localize the unknown coefficients where it is known that a more detailed reconstruction can be achieved *[O. M. Bucci et al. IEEE Trans. Geosci. Remote Sens., 39, 2527-2538, 2001]* or can be used to progressively increase the accuracy of the image by adaptively enlarging the set of unknown coefficients [*R. Scapaticci et al., IEEE Trans. Antennas Propagat., 60, 3717-3726, 2012*].

Inspired by such a framework, this communication presents a new inversion method that exploits the advantages of multiresolution representations in the framework of hybrid modified gradient (MG) finite element inversion (FEM) methods. These latter methods have recently gained an increasing attention, since they offer the capability of handling inverse scattering problems in non-conventional configurations (i.e., those for which no analytic expression of the Green's function of the scenario is available), while preserving the effectiveness (in terms of imaging reliability) of MG methods. The MG-FEM method considered in this work is the recently contrast source FEM inversion method [*A. Zakaria et al., Inv. Probl. 26, 115010, 2010*].

Clearly, the main issue in introducing a wavelet inspired regularization framework in MG-FEM is the impossibility of directly exploiting discrete wavelet transforms on grids that are not regular (and whose discretization is not a power of two). To overcome this limitation, a set of hierarchical basis functions for the 2-D FEM, which may be interpreted as a sort of generalization of the "lazy wavelet" scheme, successfully applied to method of moments (MoM) problems [*F.Vipiana et al., IEEE Trans. Antennas Propagat. Vol.53 n.7, 2005*].

The detailed description of the proposed multiresolution FEM-CSI method will be given, together with an assessment of its performance against synthetic and experimental data.