A Parallel MLMDA-Based Direct Integral Equation Solver

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Fast direct integral equations methods for analyzing time-harmonic scattering phenomena constitute an active area of research in the CEM community. These solvers present an interesting alternative to FMM-based iterative schemes for problems that are inherently ill-conditioned and/or involve many excitations. Present fast direct methods leverage the low-rank (LR) nature of blocks of the LU factors or inverse of the method of moment (MoM) matrix (if and when this property manifests itself). In essence, they leverage the limited number of degrees of freedom of the fields radiated by sources in the presence of the scatterer (or portions thereof), observed across a finite region of the scatterer (whenever the concept makes sense). LR compression strategies do not lead to low-complexity solvers for electrically large concave structures. This fact was demonstrated in [H. Guo et al., Antennas and Wireless Propagation Letters, IEEE, vol. 11, 2012], where blocks of LU factors that proved incompressible by LR methods were re-compressed by MLMDA/butterfly methods. Somewhat unexpectedly, this study showed that MLMDA/butterfly scheme produced compression above and beyond that provided by LR schemes, requiring only $O(N \log^2 N)$ final storage irrespective of the nature or electrical size of the scatterer. The practical applicability of the scheme however was severely hindered by the fact that the MLMDA/butterfly scheme acted as an add-on to a LR solver.

Here, we report on a new direct integral equation solver by that utilizes MLMDA/butterfly schemes for compressing the LU factors of a MoM matrix. When compared to its predecessor, the new solver has three important features of note. (i) The solver entirely bypasses the LR compression step of its predecessor and operates directly on butterfly-compressed blocks. The latter is achieved using new schemes for rapidly adding and multiplying butterfly-compressed operators. (ii) The solver executes in parallel, using a hybrid OpenMP-MPI strategy to accelerate the hierarchical inverse/decomposition of the MoM matrix. Despite the inherently sequential nature of this operation, the solver exhibits excellent scaling properties up to several hundred processors. (iii) The solver applies to large-scale 3D (as opposed to 2D) analysis and was implemented to invert a combined field (as opposed to an electric field) integral operator. The solver is capable of inverting MoM matrices that discretize combined field integral equations modeling scattering from electrically large structures involving millions of unknowns on a small cluster using just a few hours of CPU time.