Translation Operator Based on Gaussian Beams

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The fast multipole method (FMM) for the Helmholtz equation employs a plane-wave expansion involving a diagonal translation operator to compute the interactions between source and receiver groups (R. Coifman, V. Rokhlin, and S. Wandzura, IEEE Antennas and Propagation Magazine, 25, 7-12, June 1993). Even for large groups, the standard translation operator in three dimensions is typically not sufficiently localized to allow any of the plane-wave translations to be neglected. In other words, when computing the field in a receiver group due to sources in a source group, one must include plane waves propagating in all directions. The present paper derives a directional translation operator based on Gaussian beams that requires only plane-wave translations in a cone centered on the direction from the source group to the receiver group. The cone angle depends on the shape of the source-receiver geometry as well as on the diameter of the source and receiver regions.

The Gaussian beams (often referred to as complex source-point beams) used in the present paper were discovered by Deschamps (G.A. Deschamps, Electron. Lett., 7, 684-685, 1971), who found that the free-space Greens function in the frequency domain satisfies the wave equation even when the source point is in complex space. Gaussian beams are used in scattering theory and as basis functions in exact expansions of general solutions of the wave equation.

The present paper begins with a complex-space extension of the standard plane-wave Gegenbauer formula (the plane-wave Gegenbauer formula is the formula obtained by inserting a plane-wave expansion of a spherical Bessel function into Gegenbauers addition theorem.) The Gaussian translation operator and the associated sampling theorems emerge straightforwardly from this formula. The sampling theorems makes it possible for the Gaussian translation operator to achieve any desired accuracy. Due to the directionality of the Gaussian beams, the required sampling rate depends not only on the source and receiver group diameters, but also on the actual positions of the sources and receivers within these groups. The theory is validated through numerical examples.