A Well-Conditioned Volume-Surface Field Integral Equation (VSCFIE) for Inhomogeneous Cylindrical Scatterers with High-Electrical Contrasts

Luis J. Gomez*, and Eric Michielssen
Radiation Laboratory, University of Michigan, Ann Arbor, MI 48109, USA

Volume integral equations (VIEs) are commonly used to analyze scattering from inhomogeneous dielectric objects. Unfortunately, when applied to high-contrast objects, their discretization calls for very fine meshes and results in ill-conditioned systems of equations. This severely limits the utility of VIEs in modeling electromagnetic phenomena germane to a host of biological and geophysical applications.

A partial solution to this problem was proposed by Usner and coworkers, who preconditioned the VIE using a surface integral equation (SIE). (Usner et al., *IEEE Trans. Antennas Propagat.,* vol. 54, pp. 68-75, 2006.) First, they subdivide the object into subscatterers each having a small (controlled) ratio of maximum to minimum permittivity. Next, they wrap each subscatterer in equivalent electric and magnetic surface currents and model interactions between subscatterers using SIEs; this procedure allows for an artificial increase in the effective permittivity of the "background medium" in which each subscatterer's polarization currents radiate. Finally, all surface and volume currents are determined by numerically solving a system of volume-surface integral equations (VSIEs) consisting of coupled combined field integral equations (CFIEs) and VIEs. The VSIE somewhat alleviates the need for very fine meshes. However, if the electrical size expressed in wavelengths of the object (i.e. $\omega\sqrt{\mu_0\varepsilon_0\varepsilon_{r\max}}\,d$) is fixed, the condition number of the matrix resulting from discretization of the VSIE grows linearly with the object's maximum effective permittivity ($\varepsilon_{r\max}$). Furthermore, with the maximum effective permittivity ($\varepsilon_{r\max}$) and mesh fixed, this matrix' condition number is inversely proportional to the object's electrical size.

Here, we propose a method that alleviates both these problems. We start from a subdivision of the object into subscatterers inspired by that used by Usner and coworkers, and introduce equivalent electric and magnetic currents on their surfaces. Next, we solve for the surface and volume currents using a system of Volume-Surface Integral Equations (VSIE) consisting of coupled discrete CFIEs and new, combined VIEs. For each subscatterer, a combined VIE is constructed by judiciously adding contributions due to the currents exterior to it and propagating in the "background medium". Numerical data obtained by analyzing time-harmonic TE scattering from various 2D layered cylinders suggests that discretization of the new VSCFIE yields matrices with condition numbers that are unaffected by the object's maximum permittivity ($\varepsilon_{r\max}$) and electrical size.