Multilevel Direct Solver for the Generalized Equivalence Integral Equation

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A novel multilevel direct solver based on the recently proposed generalized equivalence integral equation (GEIE) for scattering by impenetrable essentially convex objects (A. Boag and V. Lomakin, EuCAP 2012) is presented. The solver relies on the high compressibility achievable for the discrete forms of the GEIE that are obtained, for example, via the method of moments (MoM) using sub-sectional (subdomain) basis and testing functions.

Using the generalized equivalence principle, the interior of the scatterer is replaced by a lossy impenetrable object in contrast to the free space dictated by Love's equivalence principle, which is used to derive the conventional electric field integral equation (EFIE) and combined field integral equation (CFIE). While with the EFIE and CFIE direct line-of-site interactions exist between all subdomains via the free space Green's function, with the GEIE the line-of-sight interactions between distant subdomains of convex geometries are practically eliminated, and the problem's dimensionality is essentially reduced. In the reduced dimensionality problem, the MoM matrix blocks describing the interactions between subdomains of typical length size R have $\mathcal{O}(kR)^{d-2}$ rather than $\mathcal{O}(kR)^{d-1}$ degrees of freedom (DoF) (d being the problems dimension). For two-dimensional ($d=2$) scattering by essentially convex shapes, a compression to $\mathcal{O}(1)$ unknowns can be achieved by using a lossy circular cylinder in the GEIE-based formulation, at the cost of computing a slightly more complicated modified Green's function of the circular cylinder (via the Mie series).

Based on the high compressibility of the GEIE-MoM linear systems of equations, a fast matrix compression and solution scheme is developed. A multilevel rank revealing procedure is applied to the matrices describing the interactions of subdomains of hierarchically growing sizes with the rest of the scatterrer. At each level, sets of globally interacting and non-interacting modes are computed and, using the Schur's complement technique, the problem is reduced to that of solving a system of equations for the interacting ones only (Y. Brick and A. Boag, IEEE Trans. Ultrason. Ferroelectr. Freq. Control, 58, 2405-2417). The non-uniform grid (NG) field representation is used to accelerate both the rank revealing stages and the evaluation of the interactions between the modes. This multilevel NG-accelerated algorithm combined with the high compressibility of GEIE matrices, sums up to a fast direct solver.