

## **Limitation on Far-Field Super Directivity Using Transformation Optics**

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Transformation electromagnetics has received a lot of interests in recent years. It has found potential applications in several major areas of electromagnetics: propagation, waveguiding, scattering, and radiation. Using transformation electromagnetics, it is possible to manipulate the electromagnetic waves to achieve novel effects. For propagation and waveguiding, transformed structures allow waves to propagate and bend without suffering from reflections. Regarding scattering, both monostatic and bistatic scattering can be reduced. In this paper, we analyze a spherical core-shell structure which can achieve arbitrarily large directivity. The structure was predicted and obtained from the proper coordinate transformations. A large virtual aperture can be projected to free space with a small physical dimension, subsequently leading to a large directivity. We investigated the problem by finding the transformed constitutive tensors and solving the equivalent problem in the core-shell configuration. Using the Riccati-Bessel functions, we can represent the field components with Debye potentials and subsequently solve for the fields in all regions. We applied the formulation to several cases of dipole arrays within the shell, corresponding to both free-space and halfspace problems in the virtual space. Both the near field and farfield phenomena were investigated. The relationship between the beamwidths and transmission coefficients for different loss values has been studied and an estimation for the cutoffs of the transmission coefficients was provided. Overall, the loss is linked directly to the farfield resolution in terms of the available angular harmonics. It is found that the deterioration of beamwidth changes rapidly with the increase of loss tangent in the shell medium. Even though one can theoretically construct an antenna aperture with arbitrarily high directivity in a small package, based on the specific transformation used here, there is a practical limit as to how strongly a large aperture can be compressed into a small one. It can be reasoned that if a moderate loss tangent value can be reached, a virtual aperture gain factor of at least two should be achievable.