A New Approach for Accurate Electrostatic Green's Function Computation in Planar Layered Media Based on Higher-Order Finite Element Method

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The electric performance verification of high-speed interconnects in the current and emerging integrated circuit technologies requires accurate extraction of capacitance matrices. This is accomplished via solving the pertinent electrostatic boundary value problem expressed by the linear integral equation of first kind. In case of interconnects embedded in planar layered media the unknowns are both the electric charge density on the conductor surface and the polarization charges at the dielectric interfaces. In order to substantially reduce the corresponding discrete problem size the unknown is restricted to the conductor surface via formulating and computing the electrostatic Green's function which accounts for the boundary conditions present at the interfaces of the planar layered media.

In this paper we arrive at a compact closed-form expression which accurately calculates the spatial-domain electrostatic Green's function for planar layered media via solving the corresponding spectral-domain ordinary differential equation (ODE) using higher-order finite element method (FEM) with quadratic shape functions (QSF). The mathematical formulation begins with Green's function for Poisson's equation in three-dimensional cylindrical coordinate system. Due to the azimuthal symmetry of the problem the Green's function is invariant in the ϕ direction. Application of Fourier-Bessel transform yields a spectral-domain ODE. The solution of the ODE is sought numerically via the FEM with linear shape functions (LSF) and QSF. Taking into consideration the boundary conditions at the dielectric interfaces, the formulation methodology flows such that the spectral-domain Green's function is expressed in a mathematical form where the inverse Fourier-Bessel transform can be performed analytically to yield its space-domain counterpart. Usually approximations are performed in the spectral-domain in order to facilitate the computation of the inverse Fourier-Bessel transform (K. S. Oh, D. Kuznetsov, and J. E. Schutt-Aine, IEEE Trans. Microwave Theory Tech., vol. 42, no. 8, pp. 1443-1453, 1994).

Extensive numerical studies have been conducted to show the versatility of the developed methodology in terms of accuracy versus other methods such as the finite-difference based method in (A. Cangellaris and L. Yang, IEEE Trans. Magn., vol. 37, no. 5, pp. 3133-3136, 2001) and (V. I. Okhmatovski and A. C. Cangellaris, IEEE Trans. Antennas Propag., vol. 50, no. 7, pp. 1005-1016, 2002).