Regularized Graph Laplacians for Hierarchical Spectral Partitioning

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Several integral equation formulations in Computational Electromagnetics require the partitioning of a surface manifold into sub-manifolds/macro-cells. An incomplete list of examples includes domain decomposition integral equation methods, hierarchical quasi-Helmholtz decompositions, and characteristic/synthetic basis function methods. The need for a hierarchical partitioning of the domain presents several problems in practice. When topologically complex structures have to be handled, very often it is necessary to detect the global topological loops, to properly handle the multi-scale nature of the problem, and to balance the memory loads when the algorithms are implemented on a parallel architecture.

An effective, although usually expensive, way of obtaining a partitioning of a meshed manifold is to rely on the spectral properties of the mesh associated graph Laplacian. Such an approach, known in literature as spectral partitioning, offers a robust partitioning strategy that works well even for very complex, folded, and/or non-simply connected geometries. In many of these cases, partitioning techniques based on quad/oct-tree schemes fail dramatically, jeopardizing the performance of any subsequent method relying on the partitioning.

Unfortunately, standard spectral partitioning methods are very expensive since they rely on the spectral analysis of the Laplacian which is a very ill-conditioned, and thus often intractable, operator. Several schemes to regularize the Laplacian (like hierarchical bases approaches or geometric multi-grids) rely on domain partitions to achieve the regularization. For this reason they cannot be used in this context since they would require the manifold partitioning which is instead the final goal of the procedure.

This contribution will introduce an innovative approach to the problem. The graph Laplacian will be regularized, by the scheme we are proposing, in an analytic way, by leveraging on fractional Sobolev norm realizations obtained with integral operators and by linking the Laplacian and the operators with suitably chosen mapping matrices that would connect, in the correct way, the underlying discretization spaces. In practice this will result in a multiplicative preconditioner for the graph Laplacian that, as will be detailed in the talk, will be easily obtained from matrices currently available in any standard EM boundary element implementation. Numerical results will corroborate our theoretical developments and will show the practical impact of our new technique on several realistic cases arising from applications.