Quasi-local transmission conditions for domain decomposition methods applied to scattering problems

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Scattering problems by electrically large objects are of wide interest in many fields. The numerical computation of such problems remains limited by computer resources, due to the large number of unknowns, especially when inhomogeneous materials are present. Domain decomposition methods (DDM) are particularly attractive for the solution of large finite elements problem. It is decomposed into several coupled sub-problems which can be solved independently, thus reducing considerably the memory storage requirements. Moreover, DDM are intrinsically well suited for numerical implementation on parallel computers.

The key point of these methods is how the sub-domains are coupled to each other, i.e. which transmission conditions are imposed on the interfaces between two sub-domains. The most commonly used transmission condition in electromagnetism is the one proposed by Després in [B. Després, SIAM, 197-206, 1993] or higher order ones [see e.g. M. Gander et al., SIAM JSC, 24, 38-60, 2002]. But these transmission conditions are local (i.e. expressed in terms of tangential differential operators) and generally lead to arithmetic convergence.

In this paper, we propose new transmission operators achieving exponential convergence. Using the general theorical framework developed in [F. Collino et al, Comp. Methods Appl. Mech. Engrg., 148, 195-207, 1997], we introduce linear positive symmetric and injective operators T from $H^{s}(\Sigma)$ into $H^{-s}(\Sigma)$ for some $s > 0$. Under the assumption that $T = \Lambda\Lambda^{*}$, $\Lambda \in L\left(L^{2}(\Sigma), H^{-s}(\Sigma)\right)$, and Λ is isomorphic, it is proven that the DDM algorithm converges. Moreover, if $s = 1/2$, then the convergence is geometric. This last condition prevents us from defining T with the help of local operators. To build explicit operators satisfying all the required conditions, we use nonlocal operator of the form $-div_{\Sigma}(\mathrm{K}\nabla_{\Sigma})$ where K is an integral operator with kernel $K(x, y)$ which should be a pseudo-differential operator of order -3/2. As suggested by Riesz potentials [E. Stein, Princeton Math. Series, 30, 1970], we use a kernel of the form $|x - y|^{\delta}$, where δ depends on the dimension of the space. Moreover, to avoid fully nonlocal operators, leading to full matrices after space discretization, we localize K around the diagonal $|x - y|$ by a smooth-enough cut-off function. Analytical research and numerical experiments will be presented to show how to tune the different parameters of the method to optimize the convergence rate.