Evaluation of the Sommerfeld Integral for a Dipole Over a Layered Earth Model

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This paper revisits the classic Sommerfeld integral problem of determining the fields radiated from a dipole placed over a half-space. The approach to solving this problem has a wide range of applications from microstrip antenna design to VLF radio wave propagation. Analytic expressions are derived to determine the field solutions for a set of geologic cases by asymptotically evaluating the spectral domain integral using the saddle-point method. A significant advantage of developing a set of analytical models over other methods such as FDTD is the physical insight gained from separating surface and lateral wave terms from radiated field expressions while enabling analysis with minimal computational overhead. A primary focus of this work is to examine the impact of singularities within the complex plane on the evaluated integrand as their existence and location can lead to numerical instability when integrating over an arbitrary range of scan angles and therefore care must be taken when choosing a path of integration. It is well known that the location of poles in the complex plane are associated with surface wave modes which can be accounted for by modifying the saddle-point approach as required. In addition, under certain conditions a branch point, which is physically realized as a lateral wave mode, may be located in close proximity to the saddle-point and integration along a branch cut must be considered. In order to understand the conditions under which surface and lateral waves are produced a set of geologic case studies are analyzed including: 1) a dipole located in air above the earth and 2) a dipole located within water beneath an ideal atmosphere. The analytical expressions derived for these cases are presented in this paper along with field pattern plots and a discussion of the key results.