A Hierarchical Scheme Based on the Generalized Method of Moments

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Hierarchical and multiresolution basis function schemes have been successfully employed as preconditioners for solution of the Combined Field Integral Equation (CFIE) in the context of Method of Moments (MoM) solvers. A hierarchical approach is attractive because it reduces susceptibility of the descretized CFIE-MoM system to low-frequency breakdown, and also acts as an effective preconditioner for iterative solvers, significantly speeding the rate of convergence. In past efforts, multiresolution basis sets have been constructed using hierarchical, conformal surface tessellations, most commonly on triangular meshes with corresponding hierarchical embeddings of Rao-Wilton-Glisson-type basis functions in linear combinations or wavelet-type macro-bases (see e.g. Vipiana, Pirinoli and Vecchi, TAP, 53, 2247-2258). These approaches have been successful, but suffer from the restriction that tessellations at a given level must be conformal with those at all higher levels, so that fine geometrical features dictate the maximum level of coarseness. Additionally, the approximation functions defined at different strata of the hierarchy must also conform in some sense, so that the types of basis functions that may be employed at different levels are necessarily closely linked and the choice of basis sets is significantly restricted.

In this work, we present a highly flexible hierarchical methodology based on Generalized Method of Moments (GMM) that removes constraints of inter-level conformality and allows significant freedom in the choice of basis set and surface description across levels. These capabilities are afforded by the GMM surface description, which is composed of a set of overlapping patches $\{\Omega_i\}$ that are decoupled using Partition of Unity (PU) functions. In the proposed method, GMM patches are successively merged to create a hierarchy of levels $l = 1, 2, 3 \cdots, N$ with corresponding hierarchical patch embeddings: $\Omega_{1,i} \subset \bigcup_j^{n_2} \Omega_{2,i,j} \subset \bigcup_j^{n_3} \Omega_{3,i,j} \subset \ldots \subset \bigcup_j^{n_N} \Omega_{N,i,j}$. Surface currents therefore form a similar hierarchy, $\mathbf{J}_{1,i}(\mathbf{r}) \subset \mathbf{J}_{2,i}(\mathbf{r}) \subset \mathbf{J}_{3,i}(\mathbf{r}) \subset \ldots \subset \mathbf{J}_{N,i}(\mathbf{r})$, where $Domain\{\mathbf{J}_{l,i}(\mathbf{r})\} = \bigcup_j^{n_{l+1}} \Omega_{l+1,i,j}$. Defining the hierarchy in this fashion removes the stricture of geometrical conformality between levels since patches at every level are decoupled. Additionally, the methodology allows independent control over surface fidelity at each level and permits great flexibility in the choice of basis sets. At the conference, we will provide details of the implementation of the method, convergence of approximations, and validation of its accuracy and utility.