The Inverse Support Problem With Far Field Data

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Many of the most fruitful theoretical and practical approaches in imaging (e.g., linear sampling, factorization method, no-response test, multiple signal classification (MUSIC)) are based on the estimation of the support or shape of the radiating or scattering objects from the available exterior field data. To implement in practice these and other related algorithms, as well as to better understand the theoretical issues of wave inverse problems in general, it is useful to address the question of quantifying how much information about the source or scatterer support is actually encoded in the exterior fields. The present research addresses this fundamental question by means of a novel probabilistic framework for inverse problems with connections to the field of compressive sensing. Particular attention is given to the basic inverse source problem with far field data. A rigorous methodology is developed that is based on the multipole expansion for estimation of the shape or support of the radiating source from far field data. Importantly, the proposed source support estimation methodology applies to both convex and non-convex supports.

Key previous contributions have addressed the estimation of the so-called minimum convex support of a far field (S. Kusiak and J. Sylvester, SIAM J. Math. Anal. 36, 1142-1158, 2005; A.D. Yaghjian, T.B. Hansen, and A.J. Devaney, IEEE Trans. Anten. Propag. 45, 911-912, 1997). If the minimum source region is convex then, of course, this estimated support coincides with the minimum one consistent with the given radiated field. However, more generally the minimum source region may be non-convex, in which case it is only a subset of the minimum convex support. This paper extends these previous developments in two main aspects. First, we show that the multipole expansion can be used to give not only the minimum convex source region but also the generally non-convex minimum source region. Second, we investigate the practical applicability of estimates of the minimum source region of a given source or target by characterizing, probabilistically, how well the minimum source regions or bounds of those regions approximate the true supports of the original sources producing the fields. We assume that the space of realizable fields associated to well-behaved, L2 sources of a given support is populated randomly (without bias). This leads to methodologies to estimate the probability of finding the original source in given regions based on the available far field data, which, in turn, allow the computation of probabilistic confidence intervals that are intrinsically associated to the estimation of the true source support from the respective far field. This also gives probabilistic meaning to the minimum source regions, which are shown to be probable bounds of the true source supports producing the observed far fields.