BI-CUBIC SPLINE INTERPOLATION FOR (COMPLEX) SOIL PERMITTIVITY

C. CLAY MARSTON
REMCOM, INC.

A comprehensive numerical algorithm capable of evaluating complex permittivity of soil for arbitrary frequency, volumetric water content, and as a function of the weight fraction of the primary solid components silt sand and clay is constructed. In the absence of semi-empirical models suitable for spanning the desired frequency range of .01-18,000 MHz, interpolative two-dimensional polynomials in frequency and volumetric moisture are prepared from the measured data . Soil permittivity surfaces as functions of frequency and moisture content at specified solid solids fractional compositions and for a number of specified soils solids compositions are created through a suppositions of pre existing polynomial fits (Dobson)in the range 1400-18,000 MHz, measured data of Campbell from the frequencies 1,2,5 and 10, 20 and 50 MHz for primary soils silt sand and clay and from measured data of Sternberg for frequencies in the range .001 - 1.00 MHz. The intermediate modeling expresses the permittivity as polynomial quadratic in the (fractional) moisture content according to the expression

$$\epsilon'(\{\nu, \bar{w}_{si}, \bar{w}_{sa}, \bar{w}_{cl}\}, m_v) = \sum_{n=0} c_n {m_v}^n \tag{1}$$

This formalism is extended to arbitrary composition by assuming that the coefficients in the above equation may be expressed as a linear combination of their compositional fractions in silt sand and clay

$$|\mathbf{c} >_\mathbf{n} = \mathbf{F}|\mathbf{a} >_\mathbf{n} => |\mathbf{a} >_\mathbf{n} = \mathbf{F}^{-1}|\mathbf{c} >_\mathbf{n} \tag{2}$$

where the column vector consists of a coefficients ($|c >_n$) of the nth power from each of the three independently modeled soils. The unknown combinatorial coefficients $|a >$ are obtained through inversion of the composition matrix F. The composite modeling of the primary soils in the intermediate range as described above extends the lower frequency range of permittivity as a function of frequency and moisture content at arbitrary soils solids compositions to 1 MHz. Since the lower frequency data of Sternberg is only available at a single value of volumetric moisture, the permittivity below 1 MHz was modeled as a scaled projection of the behavior at 1 MHz. The scaling factor is determined by the ratio of the permittivity at the lower frequency and single specified moisture to that at 1 MHz of the same volumetric moisture. The composite modeling of soil permittivity in the frequency range .01 MHz to 18 GHz and as function of volumetric moisture permits the evaluation of permittivity in a two dimensional grid corresponding to frequency and volumetric moisture for each soil considered . The techniques of bi-cubic spline interpolation permits the generation of a second derivative matrix which can then be used to evaluate the function at arbitrary arguments (frequency and moisture) within the two dimensional grid. Whereas the above bi-cubic spline mechanism is restricted to a specific soil composition, a final (localized) universality is achieved by assuming again that the values of the permittivity at each grid point are linear function of soil component gravimetric fractions

$$|\epsilon(\mathrm{i},\mathrm{j}) > = \mathbf{F}|\mathbf{a} >_{(\mathbf{i},\mathbf{j})} => |\mathbf{a} >_{(\mathbf{i},\mathbf{j})} = \mathbf{F^{-1}}|\epsilon(\mathrm{i},\mathrm{j}) > \tag{3}$$

where $\epsilon(i,j) >$ consists of the values of permittivity at a specified grid locus for each of the three soil textures defining the rows of the fractional matrix F. The selection of the three soil compositions for the construction of the coupled set of equation above is according to silt content in the soils solids phase diagram. Thus the soils may arranged in groups of three according to their silt content. Continuity across silt boundaries was achieved by first creating the combinatorial coefficients a¿ in the above equation for a given zone and then generating the permittivity grid corresponding to both endpoints of the border with the next higher silt content. The two border endpoints, dictated by the zone of lesser silt content, are then used as two of the defining configurations of the adjacent zone of higher content such that the adjacent zones have two common defining configurations. The permittivity of a soil of arbitrary composition and at a desired frequency and moisture composition is calculated by first identifying the appropriate zone according to its silt content which identifies the corresponding combinatorial coefficients —a¿ which are then used to construct of the full (frequency and moisture)grid appropriate for the indicated texture. The techniques of bi-cubic spline interpolation are then invoked to generate the second derivative matrix which is then used to compute the permittivity of the soil of know solids composition at the desired frequency and moisture content according to the expressions

$$\epsilon'(\nu, m_v, s, c, cL) = \sum_{n=0}^{3}\sum_{m=0}^{n-m} c_{mn}\xi^m {m_v}^{n-m}$$

$$\epsilon''(\nu, m_v, s, c, cL) = \sum_{n=0}^{3}\sum_{m=0}^{n-m} c''_{mn}\xi^m {m_v}^{n-m} \tag{4}$$

REFERENCES

[1]Campbell, J.E., Dielectric Properties And Influence Of Conductivity In Soils At One to Fifty Megahertz, Soil Science Society American Journal 54(3),332(1990).

[2] Dobson, M.C., Ulaby, F.T., Hallikainen, M.T., and El-RAyes, M.A., Microwave Behavior Of Wet Soil, Part I, IEEE Transactiona on Geosacience and Remote Sensing 23(1), 25 (I985).

[3] Dobson, M.C., Ulaby, F.T., Hallikainen, M.T., and El-RAyes, M.A., Microwave Behavior Of Wet Soil, Part II, IEEE Transactiona on Geosacience and Remote Sensing 23(1), 35 (I985)

[4]Sternberg, B.K., Levitskaya, T.M., Electrical Parameters Of Soils In Frequency Range 1 kHz To 1 Ghz Using Lumped Circuit Methods, Radio Science 36(4),709 (2001).