Proper Interpretation of the Shannon Channel Capacity for the Vector Electromagnetic Problem

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The Shannon Channel Capacity theorem has been used to specify the maximum information rate that can be sent over a channel. There are various forms of the Shannon Channel capacity theorem as Shannon described them in his original paper. The point here is that we must use the basic form of the theory as he presented and not the derived forms that relates to the power. The basic form of the theorem states that *to distinguish between M different signal functions of duration T on a channel, we can say that the channel can transmit* $\log_2 M$ *bits in time T. The rate of transmission is then* $\log_2 M/T$. *More precisely the channel capacity may be defined as* $C = \lim (T \to \infty) \log_2 M/T \approx 2BN$, *where B is the one sided bandwidth of the signal and N is the number of effective bits in which a signal is decomposed into.*

However, in electromagnetics and signal processing most of the authors use the power form and relate it to the signal to noise ratio by transforming the levels in the amplitude of the signal to the square root of the power! Hence, the form of the equation that is commonly used is related to the signal power over the noise power at the receiver, by transforming the above equation to be able to distinguish a signal in the presence of background noise. This can not always be justified as one knows from simple undergraduate circuit theory that the power cannot be computed from only the voltage or the current unless one is dealing with purely resistive circuits. Particularly in antenna problems when one is dealing with a possible near field scenario, where we may have a reactive component of the power. We need to now how that plays in the channel capacity theorem in the form that is currently being used by most practitioners. This problem becomes more quixotic when one approaches this issue from a random variable point of view, where the Fourier transform of the autocorrelation of a variable is termed the power spectral density of that variable! Therefore, if one considers a voltage or a current as the variable of interest then the Fourier transform of the autocorrelation of that apparently will provide the power spectral density! This conclusion is questionable from the Maxwellian point of view, who interestingly first introduced the concept of ensemble averaging into physics. Interestingly, Maxwell was the first person to introduce a statistical law into physics. Reading his original articles clarifies this issue.

Therefore, we need to revisit the Shannon Channel Capacity theorem and apply the proper interpretation from an electromagnetic perspective. Examples will be presented to illustrate the channel capacity between two dipole antennas located over a ground plane. The goal will be to illustrate that the interpretation of the final result differs if one uses the voltage or the power variable in the capacity equation.