Retrieving Phase Response from the Far-field Power Spectrum of the Wideband Electromagnetic System by Cauchy Method

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In general, knowledge about the magnitude provides no information about the phase, and vice versa. However, for systems described by linear constant-coefficient difference equations, i.e., rational system functions, there is some constraint between the magnitude and the phase. Such a system can be expressed by a set of poles and zeros in the complex s-plane. So by estimating the power spectrum density $\left|H^2(f)\right|$ (amplitude-only data) by Cauchy Method and searching the roots of the polynomial functions, we can get two sets of poles/zeros in symmetric pairs along the imaginary axis in the s-plane. One pole and/or zero in each pair are associated with the system function $H(f)$ and the other pole and/or zero is associated with $H^*(f)$. The phase information can be obtained from $H(f)$ by choosing the right poles and zeros.

If $H(f)$ is assumed to correspond to a causal and stable system, which is the case for most electromagnetic systems, all the poles must lie in the left hand side of the s-plane. With this constraint, only half of the poles will satisfy this constraint and will be used to construct the system response $H(f)$. Another key constraint is that assuming the right zeros are selected, there should be a linear phase difference between the reconstructed and the actual phase functions. Because for the LTI (linear time-invariant) system, changing the impulse response of the system by a time shift does not change the transfer function of the original system, except that the phase spectrum is modified by a linear phase function. So given a particular amplitude response, any phase function with a linear phase difference to the actual phase should fall into the solution set, and the slope of the linear phase difference corresponds to the time delay. To apply this constraint, at first we need to split the whole frequency band into several small sub-bands with certain overlaps. Secondly we apply the Cauchy method to every sub-band and get all the possible phase functions for each sub-band. There should be a linear phase difference between the actual phase (which we do not know) and for both the right candidates for sub-bands 1 and 2. Since these two sub-bands are overlapping, the phase difference in the overlapping part between the reconstructed phase functions in sub-bands 1 and 2 should also be linear. So we can just list all the combination of phase functions for sub-bands 1 and 2 and find the most linear phase difference, then the corresponding phase functions are the desired ones. Similarly, we can choose the phase function for sub-bands 3, 4… by observing the overlaps. Finally, all the phase functions for all the sub-bands are assembled together and the phase response for the whole frequency band is generated. To compare the linearity in the overlapping band, we apply linear regression to the sample points in the overlapping frequency range. Sometimes we may get purely imaginary poles and zeros which simply scale the system response. They will not affect the phase functions thus can be ignored.

Several Numerical Examples, including a horn antenna, a patched antenna and a hollow resonance box are presented. The difference between the actual phases and the overall estimated phases are very linear. Also in time domain, the reconstructed system response is just the shifted version of original ones except some ignorable distortion at the peaks.