Transient Plane-Wave Reflection from an N-layer Medium by the Method of Subregions

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The transient plane-wave field reflected from an N-layer planar stack of materials is shown to consist of a series of late-time responses of subregions of the stack. Each of these late-time responses exist during time periods corresponding to the excitation of the different layers of the N-layered medium. This is shown beginning in the frequency domain, where the reflection coefficient, $\Gamma_N(s)$, is determined as both a ratio of functions and a series of terms. Each of the terms in the series representation are shown to correspond to the late-time response for a subregion of the problem of interest, with the k-th subregion consisting of the first k-material layers backed by a semi-infinite region with the material parameters of region k+1.

Using a recursive form of the numerator and denominator from the ratio representation of the reflection coefficient, a reduced reflection coefficient is defined for each subregion of the material stack. The k-th reduced reflection coefficient is found as the difference between the reflection coefficient for a subregion consisting of k-layers and the reflection coefficient for a subregion of k-1 layers terminated by a material with the properties of layer k and are thus given by $\delta_k(s) = \Gamma_k(s) - \Gamma_{k-1}(s)$. The properties of this quantity prove useful in obtaining the series representation of the reflection coefficient, as well as showing that the subregion responses are turned on and off at times corresponding to the observation of different subregion excitations. This allows the series representation of the frequency domain reflection coefficient to be obtained as the interfacial reflection coefficient for the first interface plus the sum of the reduced reflection coefficients for each subregion.

Utilizing an inverse Laplace transform, the series representation of the frequency domain reflection coefficient allows the transient response of a layered material to be shown as consisting of the late-time responses of each subregion, where each subregion response is turned on and off at times corresponding to excitation of the various material layers. This result can be used with time-domain methods in determining the properties of layered materials, particularly those methods which utilize natural resonance representations.