Time-Domain Reflection from a Lorentz Medium Half-Space

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Simple electromagnetic materials are often modeled as having constitutive parameters that are constant with frequency. More complicated materials with frequency-dependent parameters are often represented using the Debye model, which is valid for many liquids at moderate frequency. For materials that exhibit resonances, or have parameters that vary rapidly with frequency, the Lorentz model is becoming increasingly more popular, especially when dealing with signals with content at optical frequencies. The propagation and reflection of transient pulses by these media is of particular interest.

It has been shown previously that the transient field reflected from a Lorentz medium half-space can be represented as an infinite sum of fractional order Bessel functions (Gray, Kenneth G, *Proceedings of the IEEE*, Vol. 68, No. 3, pp. 408-409, 1980), (Stanić, B.V., et. al., *J. Phys. D: Appl. Phys.*, Vol. 24, No. 8, pp. 1245-1249, 1991). In order to gain more physical insight into the physical behavior of the problem, a closed form solution has been formulated which has no infinite sums. The solution is obtained by taking an inverse Laplace transform of the frequency-domain reflection coefficient. The frequency-domain reflection coefficient is manipulated in such a way that it can be represented as a product of functions for which, individually, the inverse Laplace transform can be found analytically. This leads to a solution containing a finite number of convolutions of Bessel functions and exponentials which gives a clearer picture of the behavior of the reflection coefficient. Parameter and incidence angle choices determine whether the convolution terms contain ordinary or modified Bessel functions. In order to obtain a complete solution to the problem, different sets of parameters are chosen which are representative of all the possible combinations of Bessel functions and modified Bessel functions.

The results obtained using the closed-form solutions are verified through comparison with an inverse fast Fourier transform of the frequency-domain reflection coefficient.