A 3-D Spectral Element Time-Domain (SETD) Method for Electromagnetic Wave Problems

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The finite element time-domain (FETD) method has recently emerged as an alternative technique to analyze the characterization of electromagnetic devices. It has superior versatility in modeling both complex structures and materials. In most FETD methods the inverse of a large sparse mass matrix is required in each time step.

The spectral element method (SEM), based on Gauss-Lobatto-Legendre (GLL) polynomials, has the demonstrated advantages of a high-order accuracy and its block-diagonal mass matrix, and is, thus, a very efficient solver. This method can be considered as a special class of the general finite element method (FEM), with the choice of nodal points and quadrature integration points being different from the conventional FEM.

In this paper, we developed a 3-D spectral element time-domain (SETD) method to solve the first-order Maxwell's equations as well as their second-order wave equations. Galerkin method is used for spatial discretization, and a fourth-order, four-stage Runge-Kutta scheme is employed for the time integration. Because the basis functions are orthogonal and because the GLL quadrature is used, the mass matrix is block-diagonal and thus it can be easily inverted with little cost. After we obtain the inexpensive inversion of the mass matrix, it is moved to the right-hand side and multiplied by stiffness matrices and the resulting matrices will be still sparse. Finally, only a trivial vector-matrix product is required in each time step.

To account for open boundary problems in the first-order formulation of Maxwell's equations, we employed perfectly matched layers (PML) as an absorbing boundary condition. We adopt the pseudospectral time-domain (PSTD) method to simplify the PML treatment inside the SETD method. Several numerical examples illustrate the efficiency of the SETD method. The results obtained by the first-order formulation of Maxwell's equations are also compared with those obtained by the the second-order formulation.