

The FMM-Accelerated Fast Prolate Pseudospectral Method

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Abstract

The pseudospectral method utilizing prolate spheroidal wave functions (PSWFs) as basis functions has been shown to possess strong advantages over the conventional pseudospectral methods based on trigonometric and orthogonal polynomials and related functions. These include higher accuracy using smaller number of unknowns for problems involving bandlimited functions; the PSWFs are a better tool as compared to the orthogonal polynomials for representing bandlimited functions, which are ubiquitous with wave-phenomena encountered in many real-world physics and engineering applications. The prolate pseudospectral method also affords a more uniform spatial collocation grid, which not only provides better resolution near the domain center but also leads to larger stable time-steps in the explicit solution of time-dependent problems.

However, the spectral differentiation and interpolation steps of the prolate pseudospectral method involve matrix-vector products, which, if evaluated directly, entail $O(N^2)$ memory requirement and computational complexity (where N is the number of unknowns utilized for discretization and interpolation). In this work we show that the Fast Multipole Method (FMM) can be used to reduce the memory requirement and computational complexity of the prolate pseudospectral method to $O(N)$. Example simulation results are presented to demonstrate the speed and accuracy of the resulting fast prolate pseudospectral solver.

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