Analysis of Radiation and Surface Wave Modes of a Layered Dielectric Rod

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In this paper, a complex analysis approach is utilized to gain physical insight into radiation modes and surface wave modes of a circular cylindrical dielectric rod immersed in a surrounding dielectric region. Surface wave propagating modes that are confined to the dielectric rod are well known using the concept of total internal reflection. However, in order to understand all of the propagating modes, one also needs to consider the radiation modes. Unlike the surface wave modes which exist only for discrete eigenvalues, these radiation modes form a continuous spectrum.

The physics of this problem involves a layered dielectric circular cylinder aligned axially and infinitely in the z-direction with permittivity greater than that of the surrounding medium. The longitudinal invariance of this geometry leads to the use of an axial Fourier transform as well as employing cylindrical harmonic expressions for the scattered field representations.

The procedure to solve this problem begins with construction of scattered fields, both inside the dielectric rod and throughout the surrounding medium. Next the well known representation for an impressed field in cylindrical coordinates is used to apply boundary conditions at the interface of the rod and surrounding medium to solve for any unknown coefficients. Once both scattered and impressed fields are known, the entire field is then represented as the superposition of these two expressions. However, the total field obtained here is still in the transform domain and an inversion must be performed. Deformation of the inversion integral off the real axis will identify the discrete surface wave and continuous radiation modes. Pole singularities will give rise to the surface wave modes and correspond to the propagation eigenvalues leading to non-trivial homogeneous solutions to the transform Helmholtz equations. Next the branch point singularities arising from the forced solutions to the transform domain Helmholtz equations lead to choices of appropriate branch cuts. The choice of the proper branch cut as well as the choice to close the contour in the upper half plane (UHP) or lower half plane (LHP) must be chosen such that Sommerfeld Radiation condition is satisfied. Integration along the proper branch cuts lead to the radiation modes which can be either propagating or evanescent.

In conclusion, the complex analysis method of investigation will allow physical insight into full spectrum of modes, both discrete and continuous modes, giving a complete description of the fields.