Poynting Vector and Electric and Magnetic Energy Density as Focused by a Slab with Relative Permittivity and Permeability close to –1

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Super resolution focusing of a slab with ε_r and μ_r very close to –1 applies specifically to the field E_x for an incident p-polarized line source or H_x for an incident s-polarized line source. The transfer function of the slab with index of refraction $n = -1$ is the same for all components, but the other components do not originate with delta function localization from the point (line) source and consequently do not peak at the ideal focal point of the $n = -1$ lens.

In any experiment, the measured field quantity depends on the sensor. The E_x field could be measured with a carefully designed high-impedance dipole antenna. Bolometric-type sensors could measure the magnitude of the Poynting vector. The electric and magnetic energy densities are interesting as a conceptual aid to visualize the stored energy of the fields.

Here we present patterns of the magnitude of Poynting vector and energy densities for comparison with previous calculations of the focused E_x field. These results provide new measures of the focusing of these slabs. The results show that these energy-related quantities peak slightly closer to the exit plane of the slab and have a tighter focus than the E_x field magnitude in our example.

As one example for the case of $\varepsilon_r = -1.005\text{-}0.001j$, $\mu_r = -0.99502\text{-}10^{-7}j$ and a slab of thickness d_2 = 5 wavelengths, the transverse focal width at half peak of the magnitude of the Poynting vector is 0.39 wavelengths, and the focal width at half peak in the propagation direction is 0.92 wavelengths. The comparable widths (at 3 dB from peak) for the magnitude of E_x are 0.44 wavelengths in the transverse direction and 1.19 wavelengths in the propagation direction.

The transverse width of these patterns in the focal plane is known to get smaller as the slab thickness decreases. Significant decreases in this width require slabs less than one wavelength thick with a reasonable loss term. However, there is a transition with thin slabs to monotonic decrease of field amplitudes with distance from the slab. This characteristic will be illustrated with examples.