Error, Dispersion, and an Explicit Numerical Evaluation of a Time Domain Path Integral for the Electromagnetic Field

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A few years ago an unconditionally stable, dispersionless, implicit numerical method for evaluating a full wave time-domain Feynman Path Integral expression for the electromagnetic field was introduced (R.D. Nevels, J. A. Miller, and R. E. Miller, "A Path Integral Time-Domain Method for Electromagnetic Scattering," *IEEE Trans. Antennas Propagat.*, vol. 48, No.4, pp. 565-573, April 2000). Because the mathematical expression for this path integral has Fourier transformation as its essential component, an implicit numerical solution arising from fast Fourier transformation (FFT) was a natural fit. However, it was found that it is difficult to impose the boundary conditions that account for scattering or waveguiding objects when using FFT's. An additional drawback is that FFT's are subject to Gibbs phenomena at surfaces and interfaces as well as aliasing error that is particularly prevalent at the extremities of the numerical region.

More recently a more compact Path Integral for the electromagnetic field has been derived (R. D. Nevels and J. Jeong, "The Complete Free Space Time Domain Green's Function and Propagator for Maxwell's Equations", IEEE Trans. Antennas Propagat., Nov., 2004). This result is very close to what one would expect to see as the mathematical form of Huygens' principle. An integral operator, which in this case is in spherical spatial coordinates, propagates the initial electric and magnetic field through a sequence of time steps to produce the present time field. However in this case the integral operator, known as the propagator, is not a Fourier transform and can be conveniently evaluated by an explicit method. Here we will present an explicit numerical method for evaluating this Path Integral expression. It will be shown that the numerical error is small, on the order of Δx^5 where Δx is the spatial increment of the numerical grid. Also, an expression for numerical dispersion will be derived demonstrating a remarkable property of this method, which is: even though this is an explicit method, waves propagate through the numerical lattice without numerical dispersion. A method for implementing boundary conditions on a 3-D perfect conductor will also be presented. Examples will include 3-D cases in which a plane wave scatters from dielectric and metallic objects.