Numerical Validation of the Spectral Discrete FDTD Green's Function for the Wave and Maxwell's Equations

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A $\mathcal{Z}$–transform based methodology has recently been developed (R. Kastner, "A Multi–Dimensional $\mathcal{Z}$-Transform Evaluation of the Discrete Finite Difference Time Domain Green's Function", *IEEE Trans. on Antennas and Propagation*, accepted), that applies to the 1–to–3 dimensional discrete Green's functions. The spectral–domain versions of the Green's functions for the discretized 1–D wave equation and Maxwell's equation are expressed in Equations (1) and (2), respectively:

$$\widetilde{G}_k^{\text{wave}}(\omega) = \Delta z \frac{\psi_z}{\sin\psi_z} \frac{\jmath}{2\kappa_z} e^{-\jmath|k|\psi_z}, \qquad \psi_z = \kappa_z(\omega)\Delta z,\ z = k\Delta z. \tag{1}$$

$$\widetilde{G}_k^{\text{Maxwell}}(\omega) = -\frac{\eta e^{-\jmath\frac{\Omega}{2}}}{2\cos\frac{\psi_z}{2}} e^{-\jmath k\psi_z}. \tag{2}$$

In the continuous limits $\psi_z \to 0$ of (1) and (2), the transforms of a step function and a delta function, respectively, are retrieved: $\widetilde{G}^{\text{wave}}(z;\omega) = \frac{\jmath}{2\kappa} e^{-\jmath\kappa|z|},\ \kappa = \omega\sqrt{\mu\epsilon}$, $\widetilde{G}^{\text{Maxwell}}(\omega;z) = -\frac{\eta}{2} e^{-\jmath\kappa z};\quad \kappa = \omega\sqrt{\mu\epsilon}$. Numerical implementations for Eqns. (1) and (2) are compared in Fig. 1 with straightforward FDTD computations. The source term in both cases is the limited–duration pulse $P(n) = \sin^3(\frac{\pi}{N_p}n),\ -N_p \le n \le N_p$, and $P(n) = 0$ elsewhere, whose spectrum extents into the FDTD cutoff region. One can see that the results obtained by using Eq. (1) and Eq. (2) are virtually indistinguishable from the FDTD computation, regardless of the amount of numerical dispersion. These numerical results support the analytical expressions obtained earlier.

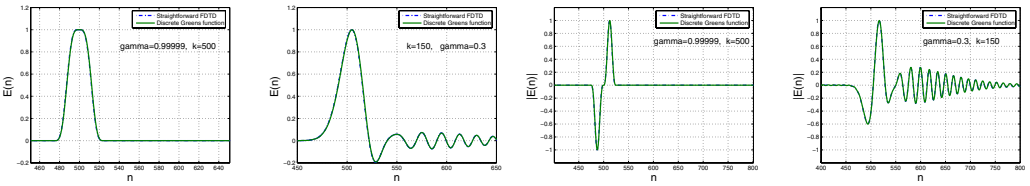

Figure 1: Propagated pulse vs. time at 500 time steps. Solid lines: Results with discrete Green's functions; dashed-dotted line: straightforward FDTD computation. Wave equation, with numerical inversion of Eq. (1): (a) - $\gamma = 0.99999$, i.e., $k = 500$ samples from the source; (b) - $\gamma = 0.3$, with $k = 150$ accordingly. ((a)–(b)) and for the the Maxwell's Maxwell's equations, with numerical inversion of Eq. (2): (c) - $\gamma = 0.99999$, with $k = 500$ accordingly; (d) - $\gamma = 0.3$, with $k = 150$ accordingly.