Second-Order PML in the Analysis of Waveguide Problems

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In a recent work (D. Correia and J.-M. Jin, *IEEE Trans. Antennas and Propag.*, vol. 53, no. 12, pp. 4157-4163, 2005) we showed how a higher-order perfectly matched layer (PML) could be formulated for a better absorption of evanescent and propagating waves in both radiation and periodic problems. In this work we present the second-order PML for the finite-difference time domain (FDTD) analysis of waveguide problems and compare its performance to the complex-frequency shifted (CFS) PML and the regular PML. The second-order PML performs better than the CFS-PML when low-frequency propagating waves are present. Both the second-order and the CFS PML's perform better than the regular PML when absorption of evanescent waves is critical. The metric coefficient for the second-order PML is given by

$$s_x = \left(\kappa_1 + \frac{\sigma_1}{\alpha_1 + j\omega\epsilon}\right)\left(\kappa_2 + \frac{\sigma_2}{\alpha_2 + j\omega\epsilon}\right). \tag{1}$$

By setting $\sigma_2 = 0$ the CFS-PML is recovered. If $\alpha_1 = \sigma_2 = 0$ the regular PML is recovered. The details on the implementation of this metric can be found in the paper cited above.

Berenger applied the CFS-PML to the FDTD analysis of waveguide problems (J.-P. Berenger, *IEEE Microwave and Wireless Comp. Letters*, vol. 12, no. 6, pp. 218-221, 2002) where he showed that the CFS-PML can perform better than the regular PML if only one mode is involved since in such case only low-frequency evanescent waves and high-frequency propagating waves are present. However, if the problem requires the absorption of both propagating and evanescent waves in the entire spectrum, the second-order PML is the best option, because of the CFS-PML limitation for the absorption of low-frequency propagating waves and the regular PML limitation for the absorption of evanescent waves. In this work we analyze parallel-plate waveguide problems by exciting not only the TM_1 mode but also the TM_2 and the TEM. Since the cutoff frequency for the TEM mode is 0 Hz, the entire spectrum contains propagating waves. In that case, the CFS-PML does not work at the lower end of the spectrum while the regular PML has a poor performance around the cutoff frequencies for the higher-order modes. The second-order PML can overcome both limitations. In the analysis of waveguide problems, it is generally not possible to predict which modes are going to be excited if no analytical solution is available. A truncation technique, such as PML, should be robust enough to absorb all modes. We use a modulated Gaussian pulse to excite TEM, TM_1 and TM_2 modes with the same amplitude and the reflection from the second-order PML is below -80 dB for the entire spectrum while the CFS-PML has a much larger reflection, around -30 dB, at the lower end of the spectrum. This clearly demonstrates that the second-order PML can provide a much higher accuracy for the FDTD analysis of problems involving waveguides.