A Markov Model for Nonuniform Random Transmission Lines

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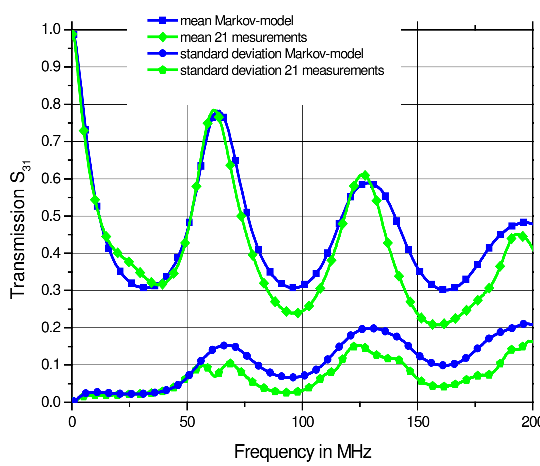
The number of transmission lines in complex systems has been rapidly increased during the last years. A today's mid-range car contains approximately two thousand wires with a total length of about 4 km. Another example is the fuselage of the Eurofighter which contains 130 cable bundles with a length of about 33 km and a weight of approximately 350 kg. In simulation and analysis cables are often assumed to be uniform, however, in real life most of the cables are nonuniform. New simulation methods and propagation models for nonuniform cables were presented in [H. Haase, T. Steinmetz and J. Nitsch, IEEE Trans. Electromagn. Compat., Vol. 46, No. 3, pp. 345-352, August 2004]. But for the application of these deterministic methods the geometric position of all wires along the cable tube must be known. In the majority of cases these data is not available. Only at the beginning and at the end of the cable tube the wires are fixed by connectors, in between the geometric positions may randomly change. These variations lead to deviations of the transmission and EMC characteristics if compared to an ideal, deterministic situation.
We will present a Markov model for a random transmission line which is built of uniform segments. The cross-section and the segment length are stochastic quantities. The segment's cross-section is chosen randomly from a finite number of possible values. The segment length is an exponentially distributed random variable. With these restrictions a Markov-model can be formulated which is described by transition densities.
The complex transmission line equations must be written as a real-valued matrix equation. With this prerequisite and the Markov characteristics of the transmission line the conservation equation of probability can be formulated, and the resulting equation can be transformed to a differential equation of the conditional probability density function for the solution vector of currents and voltages. Multiplication with the solution vector and integration over the whole vector space leads to differential equations for the conditional first and second stochastic moments. The mean, variance and covariance of voltages and currents or matrix elements of the matrizant can be computed using these moments. The stochastic characteristics of derived quantities, like scattering parameters, can be obtained by the error propagation law. Results of the Markov model will be presented and compared to measurements of a nonuniform random line made of two wires above a ground plane. In summary it can be ascertained that the Markov model is well suited to describe the stochastic scattering characteristics of a nonuniform random transmission line.



Mean and standard deviation of the magnitude of the S50-Parameter S31 (transmission) of a two wire line above a ground plane