Boundary Conditions for the Symmetry Cell in Using Finite Group Method to the Electric Field Integral Equation

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Surface integral equations are typically used in modeling electromagnetic scattering problems in free space where the numerical evaluation of the underlying equations is commonly been carried out by means of the Method of Moment (MoM). In order to minimize the computational effort it is desirable to eliminate possible redundancies if the structures involved possess a certain geometric symmetry.

The Finite Group Method (FGM) based on the harmonic analysis on finite groups (R.P. Tarasov, Comp. Math. and Math. Phys., 32, 9, 1992) offers a possibility for exploiting geometric symmetries in which the excitation is not subject to such restrictions. The main idea behind that method consists in a decomposition of the non-symmetric field quantities in their symmetric components regarding the underlying symmetry group of the considered structure. Therewith the original problem can be reformulated as a coupled set of integral equations. They are defined on a symmetry cell being the $1/N$ part of the whole domain. Here N represents the order of the symmetry group. These equations can be interpreted as a convolution on group because of the invariance of the integral operators with respect to symmetry transformations. Finally, the application of the abstract Fourier transform yields a set of approximately N independent integral equations defined only on $1/N$ part of the initial domain. Thus the computational effort can be drastically reduced by a factor depending on the underlying symmetry group. Note that the asymptotic complexity remains the same.

For the numerical evaluation of the resulting integral equations the behavior of the transformed surface quantities on the cutting edges of the symmetry cell plays an important role. Up to now the Finite Group Method has been applied to boundary integral equations for which the unknown functions need not fulfill any continuity conditions so that no further restrictions occur. We propose an extension to the electric field integral equation (EFIE) discretized by the well-known Rao-Wilton-Glisson (RWG) basis functions. For these purposes we have derived the boundary conditions for the transformed electric surface currents on the symmetry cell from the continuity conditions of their normal components. To realize these conditions within the framework of the Method of Moment we have introduced modified elements ('half' RWG-functions) on the cutting edges of the symmetry cell. This leads to additional constraints which have to be incorporated into the algebraic system of equations arising from the discretization of the surface integral formulation. Selected problems of electromagnetic scattering from perfect electric conductors with different geometric symmetry properties are numerically analyzed to validate the proposed extension of the Finite Group Method.