High-Order Hierarchical Legendre Basis Functions for Triangular Discretizations

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Several studies have indicated that with high-order basis functions faster solution convergence with respect to the number of unknowns can be obtained than with low-order ones. The high-order basis functions can be categorized as interpolatory or hierarchical. The latter ones are more flexible in the sense that they allow usage of basis functions of different orders in the same mesh and thus are available for p adaption. The drawback is that the hierarchical basis functions usually lead to worse conditioned matrix equation than the interpolatory ones. Traditionally the hierarchical high-order basis functions are based on simple power expansions and in that case the condition number of the matrix increases very rapidly with respect the expansion order. This, in turn, leads to ineffective iterative and fast solutions.

Recently, E. Jørgensen et al. (E. Jørgensen, J.L. Volakis, P. Meincke and O. Breinbjerg, IEEE Trans. Antennas Propag., 52(11), 2985-2995, 2004) proposed a new class of hierarchical divergence conforming basis functions for electromagnetic modeling. These basis functions are based on the orthogonal Legendre polynomials. With proper scaling factors these basis functions lead to significant improvement on the matrix conditioning and, hence, lead to more efficient iterative solutions, too. However, in the paper by Jørgensen et al. these basis functions were developed only for quadrilateral elements and the basis functions on triangular elements were considered as degenerate quadrilaterals with two vertices merged into one.

In this talk basis functions compatible with the aforementioned Legendre basis functions of Jørgensen et al. are developed for triangular discretizations. The idea is to first express the basis functions based on the power expansions on quadrilateral elements by the nodal and edge shape functions of a quadrilateral. Then these basis functions are transformed into triangular elements by replacing the shape functions of a quadrilateral by the shape functions of a triangle. Finally, the Legendre basis functions, on both quadrilateral and triangular elements and on mixed triangular-quadrilateral elements, are obtained as linear combinations of the basis functions based on the power expansions.