A Posteriori Error Estimator and Adaptive Mesh Refinement in Domain Decomposition Methods

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In this presentation we demonstrate the use of h-adaptive mesh refinement (h-AMR) to improve the accuracy of solution via the Domain Decomposition Method (DDM). The use of h-AMR is widely accepted to improve error convergence of the conventional Finite Element Method (FEM). Here, we use an efficient h-AMR scheme in conjunction with the DDM to solve electrically large problems with repeating sub-structures.

Conventional h-AMR improves solution accuracy by refining the geometric discretization, or FEM mesh, only in regions where error is high. A posteriori error estimates are obtained for each element and elements whose error is deemed high are selected for further refinement. The refined mesh is used in FEM solution and the method proceeds iteratively until acceptable solution error is achieved.

The DDM has been successfully applied to solve electrically large problems when the region of interest is composed of multiple repeating sub-structures. Many problems lend themselves naturally to a domain decomposition scheme; an important application arising in communications involves the analysis of large finite antenna arrays (S.-C. Lee, M.N. Vouvakis, J.-F. Lee, J. Comp. Phys., **203**, 1-21, 2005). The method requires the identification of repeating sub-structures, or building blocks, and the decomposition of the region of interest into sub-domains based on these blocks. Much of the computation associated with a block can be re-used in FEM solution, yielding a significant reduction in required resources.

Several mechanisms allow the DDM to efficiently solve large problems. The cement variables used to "glue" sub-domains together are particularly important. These variables allow non-conformity between the FEM meshes of adjacent sub-domains and thus allow each region to be meshed and refined independently. The h-AMR scheme is therefore applied only to individual building blocks, not the entire domain, saving considerable computational resources in comparison to a conventional scheme. Furthermore, refinement is only performed on those building blocks with large error; blocks with acceptable error are omitted from the procedure and prior computation re-used.

The overall DDM problem is solved via a Krylov subspace method. The convergence behavior of the iterative solver depends heavily on the enforcement of boundary conditions between adjacent sub-domains. This is accomplished here via novel higher-order transmission conditions which make use of the cement variables. In the h-AMR scheme adopted here, fast convergence using the new transmission conditions is crucial for efficient problem solution.