# A Non-uniform Grid Algorithm for Rapid Evaluation of Fields Radiated in Layered Media

Vitaliy Lomakin* (1) and Amir Boag (2)

(1) Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92122, vitaliy@ece.ucsd.edu
(2) School of Electrical Engineering, Tel Aviv University, Tel Aviv, 69978, Israel

Electromagnetic properties of objects embedded in layered media are of interest in many applications in physics and engineering. Method of moments, which is widely used for computing scattered fields in layered media, is based on evaluating fields radiated by equivalent sources distributed on/in the object of interest. The direct evaluation of the field due to $N_s$ sources at $N_s$ observers requires $O(N_s^2)$ operations. Several fast multipole method based schemes allowed reducing this high computational cost to $O(N_s \log N_s)$ operations. However, the implementation of these schemes for problems requiring the adaptivity with respect to problem geometrical features, e.g., electrically large configurations having electrically small details, is non-trivial.

This work introduces a novel scheme, referred to as layered medium non-uniform grid (LMNG) method, for fast computation of electromagnetic fields radiated by sources embedded in electrically thin layered media. The method utilizes ideas of recently introduced non-uniform grid (NG) method that is geared towards the computation of fields radiated by finite constellations of frequency/time dependent sources residing in free space. The NG method exploits the fact that the spatial bandwidth of phase/time- and amplitude-compensated fields outside a source domain is proportional to the domain's electrical size thus permitting the computation of these fields by locally interpolating from their samples on a sparse non-uniform grid. The NGTD method is adaptive with respect to the scatterer's geometrical features, e.g. can handle easily configurations that are electrically large and have fine geometrical features.

The LMNG method is based on our knowledge of the asymptotic behavior of the layered medium Green function for large and small source-observer separations. For electrically large source-observer separations, the Green function can be written as a sum of a geometrical ray and surface waves. Alternatively, the Green function can be represented as a sum of a finite number of cylindrical waves. For electrically small source-observer separation, the field is dominated by its quasi-dynamic component, viz., the spectral integral of the static limit of the spectral domain layered medium Green function. The remaining part of the field is a smoothly varying function of the spatial coordinates that can be found via a spectral integration. Alternatively, the Green function can be written as a sum of a finite number of spherical waves radiated by specially constructed complex images. All these components of the Green function are characterized by distinct propagation constants and decay rates that can be fond either analytically or numerically.

After finding an asymptotic representation for the Green function in terms of a small number of closed-form components, the fields radiated by a source domain is computed by applying the NG method to each one of these components separately and aggregating the resulting fields. Finally, to compute the fields radiated by an extended source distribution, the source domain is divided into a number of sub-domains and the interactions between these sub-domains are accounted for by interpolating from the fields computed at a set of sparse NGs. The computational cost associated with the field calculation is reduced to $O(N_s^{3/2})$ and $O(N_s \log N_s)$ operations for two- and multi-level LMNG implementations, respectively.