Model order reduction via Singular Value Decomposition of the far field operator

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The object of this paper is to show that the Radar Cross Section (RCS) of (or the far field radiated by) perfectly conducting bodies can be accurately computed by employing a reduced number of basis functions defined on their surfaces and obtained via the Singular Value Decomposition (SVD) of the far field operator.

Let $\mathbf{A}(\underline{J},\underline{r})$ be the far field radiated (or scattered) in direction $\underline{r}$ ($|\underline{r}|=1$) by the electric current $\underline{J}$ defined on the surface S of the conductor. Note that S may be non-connected if several bodies are considered. Let $\underline{\underline{A}}$ denote the matrix of the far field operator $\mathbf{A}$ written in a given set of basis functions for $\underline{J}$ (e.g. the MoM basis functions) and a given set of directions $\underline{r}$; for a given polarization, the dimensions of both sets are, respectively, N and M. The SVD of $\underline{\underline{A}}$ yields the singular values μ_q and the singular vectors $\underline{J}_q$ defined on S. It is well known that the number of degrees of freedom M that characterizes the far field is of the order of $2(kR)^2$ where k is the wave-number and R the radius of the sphere circumscribed to S. Also, $\mathbf{A}$ is a compact and regularizing operator. This entails that μ_q decreases exponentially with q for q>M. The unknown coefficients α_q of the series expansion of $\underline{J}$ in basis $\underline{J}_q$ are calculated by solving $\underline{\underline{Z}}'\underline{\alpha}= \underline{b}'$ with $Z'_{qq'}= \underline{J}_q^* .\underline{\underline{Z}}\, \underline{J}_{q'}$, $b'_q= \underline{J}_q^* .\underline{b}_q$, $1\leq q, q'\leq Q$ (the * denotes the complex conjuguate) where $\underline{\underline{Z}}$ and $\underline{b}$ are the matrix and right-hand side of the standard MoM system written on S.

If Q=N, then the "exact" current $\underline{J}$ is recovered. However, if $\underline{\underline{Z}}'$ satisfies certain conditions, then it is shown that the RCS can be accurately computed if $Q\approx M$, thus considerably reducing the computational complexity of the original problem when M<<N. Also, when N is large, the important issues pertaining to the efficient computation of the SVD of $\underline{\underline{A}}$ on the one hand, and of $\underline{\underline{Z}}'$ on the other hand, are addressed. Numerical results are presented that illustrate the pertinency of this approach.