

## Accelerating the Aggregation and Disaggregation in the Stable Plane Wave Method

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Currently, a large number of so-called Fast Multipole Methods exist. These methods accelerate the matrix-vector multiplications required for the iterative solution of the linear system of equations arising in Method of Moments based integral-equation solvers. Unfortunately, most of these methods only apply to a specific type of problem. Fast Multipole Methods based on multipole expansions of the Green function reduce both the computation time and memory requirements to  $\mathcal{O}(N)$  -  $N$  denotes the number of unknowns - for low-frequency problems, but not for high-frequency problems. Fast Multipole Methods based on plane wave expansions of the Green function reduce the computation time and memory requirements to  $\mathcal{O}(N)$  for high-frequency problem, but completely fail for low-frequency problems due to roundoff errors. Very recently, a novel Fast Multipole Method was presented that addresses both these problems, namely the Stable Plane Wave Method (E. Darve and P. Havé, Journal of Computational Physics, 197 (2004), 341-363). The Stable Plane Wave Method uses an integral representation of the Green function to approximate it by a finite sum of propagating (or homogeneous) and evanescent (or inhomogeneous) plane waves, leading to a Fast Multipole Method that (i) is numerically stable for both low- and high-frequency problems and (ii) leads to an  $\mathcal{O}(N)$  complexity for both types of problems. However, its main drawback is that the used integral representation only converges in one half-space of choice. By consequence, in practice six integral representations are needed to cover the complete space. For the propagating plane waves, all six integral representations can be calculated starting from one radiation pattern by letting the translation operator depend on the half-space. In this case, only one aggregation and one disaggregation are needed. For the evanescent plane waves, this has not yet been proven possible.

However, when considering interactions between spheres, the standard Green function has to be replaced by the multipole translation matrix (R.C. Wittmann, IEEE Transactions on Antennas and Propagation, Vol. 36, No. 8, August 1988). Just like the Green function this matrix can be written as a finite sum of propagating and evanescent plane waves. In this contribution, a very elegant way to calculate all six integral representations with only one aggregation and one disaggregation, including the evanescent part of the integral representation, will be presented. The proposed scheme relies on the use of rotation matrices and reduces the memory requirements for the aggregation and disaggregation by a factor six. For the computation time this is only valid if the maximal multipole order is larger than or equal to 2, which is true for all practical cases. It will be shown that the proposed scheme applies to both the scalar and the vectorial (Maxwell) case.