IE-FFT Algorithm for Volume Integral Equations in Electromagnetic Scattering

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Electromagnetic scattering and radiation problems that involve inhomogeneous structures find a wide range of applications such as RCS computation of material coated PEC targets, radome design and interaction of electromagnetic fields with biological media. These problems can be formulated in the form of either a differential or a volume integral equation (VIE). Among the differential equation methods, finite element method and finite-difference time domain method are most widely applied. Both of these methods employ artificial boundary condition which requires the discretization of not only the scatterer, but also the space around it. On the other hand, VIEs satisfy the radiation condition inherently because the Green's function exists in the integral equation kernel. Moreover, nontrivial backgrounds like multilayer structures can easily be included in the Green's function.

However, VIEs are largely superseded by the differential equation methods due to their high computational demands. When VIEs are solved by the method of moments (MoM), the solution of the matrix equation by a Krylov-space iterative solver, such as the generalized minimum residual algorithm, to obtain the unknown vector distribution within the scatterer requires $O(N^2)$ memory and $O(pN^2)$ computational complexity, where N is the total number of unknowns needed to characterize the volume of the scatterer and p is the number of iterations for the iterative solver to converge to the desired solution within predetermined tolerance. Even for an electrically small volumetric scatterer, this computational cost can exhaust the available resources. Hence fast solvers become essential tools to solve complex real life problems.

In attempt to expedite the iterative solution of VIEs, several methods/algorithms have been proposed and successfully applied. Multilevel fast multipole algorithm (MLFMA) (C. C. Lu, IEEE AP-S Int. Symp. Dig., July 2000, 1864-1867), and grid based methods such as conjugate-gradient (CG) fast Fourier transform (FFT) method (P. Zwamborn and P. M. van den Berg, IEEE Trans. Microwave Theory Tech., 40, 1757-1766), precorrected FFT method (X. C. Nie, L. W. Li et al., IEEE Trans. Antennas Propagat., 53, 313-320) and adaptive integral method (AIM) (Z. Q. Zhang, Q. H. Liu, IEEE Antennas and Wireless Propagation Letters, 1, 102-105, 2002) have been extended to VIEs. All these methods require $O(N)$ memory and $O(N\log N)$ solution time to solve VIEs.

In this study we propose to extend the grid based IE-FFT algorithm (S.M. Seo and J. F. Lee, IEEE Trans. Magn., 41, 1476-1479) to VIEs. The basic idea of the IE-FFT algorithm is to interpolate the Green's function by algebraically simple polynomials on a regular Cartesian grid which allows one to separate the source and observation variables. The coefficient matrix $\mathbf{G}$ stores the values of Green's function at each point on the defined Cartesian grid. Hence $\mathbf{G}$ has a Toeplitz structure, which helps to compute the interactions between well-separated elements using fast Fourier transform (FFT). Therefore, the IE-FFT algorithm is expected to perform the matrix-vector products in the iterative solver efficiently.