Transport Equations for Multilayer Random Media with Rough Interfaces

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Scattering from multilayer random media with rough interfaces is often studied using the phenomenological radiative transfer (RT) (Ulaby, Moore and Fung, *Microwave Remote Sensing*, Vol. 3, Artech House, 1986). In this approach one uses a system of transport equations corresponding to the layered random media and then one imposes the transport theoretic boundary conditions. The fundamental quantity in this approach is the radiant intensity and is not suitable for studying wave phenomena such as interference, diffraction, etc. A suitable alternative is to follow the statistical wave (SW) approach. One way to better understand the RT approach is to employ the SW approach and hence derive the equations for radiant intensities. We have carried out this task in this paper and hence clarified some issues regarding the RT approach.

The geometry of the problem consists of N layers of random media with rough interfaces. The permittivity of the j'th layer is given as $\epsilon_j + \tilde{\epsilon}_j(\mathbf{r})$, where ϵ_j is the deterministic part and $\tilde{\epsilon}_j$ is the randomly fluctuating part. The rough interfaces are described as $z = z_j + \zeta_j(\mathbf{r}_\perp)$, where z_j is a deterministic constant and ζ_j represents the random fluctuations. We assume that $\tilde{\epsilon}_j$ and ζ_j are zero mean stationary processes independent of each other. To keep our discussions simple we restrict attention to the scalar problem.

We start with a system of wave equations governing the scalar waves in the multilayer random medium. To enable multiple scattering analyses it is convenient to translate this system to a system of integral equations. The analysis for the first moment was done in Mudaliar (*APS-URSI Symposium*, 2005). The main task in this paper is on the derivation of the equations for the second moments. In the case of unbounded random media it was shown (Barabanenkov et al., *Sov. Phys. Usp.*, **13**, 551, 1971) that one can arrive at the RT equations on applying the quasi-stationary field condition to the ladder approximated Bethe-Salpeter equation. For our problem we find that we need to impose in addition the weak-surface correlation approximation. To derive the equations for radiant intensities we employ the Wigner transforms and quasi-stationary field approximation . For deriving the boundary conditions we have to identify the upward and downward travelling radiant intensities. We find that the boundary conditions hence derived are still not the same as in RT equations. One needs to further impose an approximation involving the surface correlations. Essentially this approximation requires that the wave interaction with the surface is local. In the RT approach the differential scattering cross section is an essential quantity which is used in the optical theorem to derive the extinction coefficient. On the other hand in the SW approach the propagation characteristics are influenced not only by the medium but also by the boundaries. Thus we need to impose further approximations to arrive at results similar to those of RT system. Only after implementing all these approximations one arrives at the system used in RT approach. This analysis has thus helped us to a better understand the meaning and import of the RT approach as applied to our problem.