The Pulse Designing in UWB System and a Novel Wavelet/Sinuquotient

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Introduction

In the reference [1], a generating-TDMA-signal method-Carrier Interferometry-CI is proposed. In using the Dobecci basic wavelet to detecting ultra-wideband signals in noise, each wavelet can be a UWB signal by definition [2]. There are many papers, which deal with UWB signals with wavelets such as $g(t) = e^{-jt}e^{-t^2}$ [3]. This paper gives a UWB signal generating method with the extended trigonometric function wavelet -sinuquotient. That CI method can generate a UWB signal gives light to the author to study the related pulse shape. Different from the conventional trigonometric function wavelet, the paper gives a sin (ωx)/x and (1-cosωx)/ as a novel wavelet-sinuquotient. The paper analyzes the characteristics of the sinuquotient, and discusses it as a new tool to deal with the UWB signal.

The characteristics of sinuquotient

The CI method (generating TDMA signal): $\sum_{i=1}^{N} A\cos(i2\pi\Delta ft)$, 1=<i<=N and obtaining a signal as following:

$$\sum_{i=1}^{N} A\cos(i2\pi\Delta ft) = A\times\frac{\sin(\frac{N}{2}2\pi\Delta ft)}{\sin(\frac{1}{2}2\pi\Delta f)}\times\cos(\frac{N+1}{2}2\pi\Delta ft) = A\times\frac{\sin(N+\frac{1}{2})\times 2\pi\Delta f\times t-\sin(\frac{1}{2}\times 2\pi\Delta f\times t)}{2\sin(\frac{1}{2}\times 2\pi\Delta f\times t)} \quad (1)$$

Where A = $\sqrt{1/N}\sqrt{2/T_s}$.

In designing the optimum UWB signal, we adopt a method like CI method to generate the UWB signal, which is according to FCC mask. The method is different from the former method that the parameter A is 1. It is not only simplifying processing but also giving convenience in synthesizing signals.

It is supposed that the minimum-frequency triangle function has the Δf/2 frequency. The upper frequency and the lower frequency must be integer times of Δf/2; otherwise it will generate ∞ in the pulse. The result is obtained as following (mark N×2π×Δf=ω):

$$\sum_{i=1}^{N}\cos(i\times 2\pi\times\Delta f\times t) = \frac{\sin(N+\frac{1}{2})\times 2\pi\Delta f\times t-\sin(\frac{1}{2}\times 2\pi\Delta f\times t)}{2\sin(\frac{1}{2}\times 2\pi\Delta f\times t)} = \frac{\sin(\omega t+\frac{1}{2}\times 2\pi\Delta f\times t)-\sin(\frac{1}{2}\times 2\pi\Delta f\times t)}{2\sin(\frac{1}{2}\times 2\pi\Delta f\times t)} \quad \text{(2a)},$$

$$\sum_{i=1}^{N}\sin(i\times 2\pi\times\Delta f\times t) = \frac{\cos(\frac{1}{2}\times 2\pi\Delta f\times t)-\cos(N+\frac{1}{2})\times 2\pi\Delta f\times t}{2\sin(\frac{1}{2}\times 2\pi\Delta f\times t)} = \frac{\cos(\frac{1}{2}\times 2\pi\Delta f\times t)-\cos(\omega t+\frac{1}{2}\times 2\pi\Delta f\times t)}{2\sin(\frac{1}{2}\times 2\pi\Delta f\times t)} \quad \text{(2b)}$$

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When Δf reducing, the pulse is becoming sparse, and it can be considered as only 1 pulse when $\Delta f \to 0$. We have the following formula approximately(K and P are constant): $\lim_{\Delta f\to 0}\sum_{i=1}^{N}\cos(i\times 2\pi\times\Delta f\times t)\text{->}K\frac{\sin\omega t}{t}$, $\lim_{\Delta f\to 0}\sum_{i=1}^{N}\sin(i\times 2\pi\times\Delta f\times t)\text{->}P\frac{1-\cos\omega t}{t}$ (3)

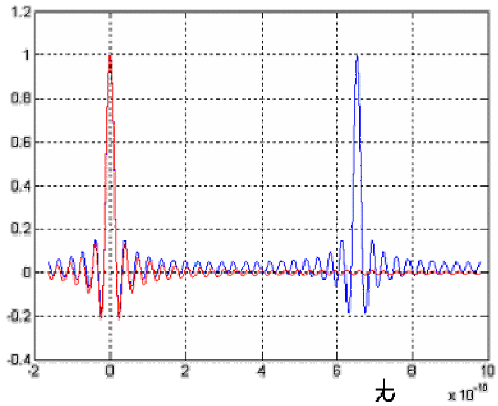


Figure 1 $\sum_{i=1}^{N}\cos(i\times 2\pi\times\Delta f\times t)$ and $\frac{\sin\omega t}{t}$

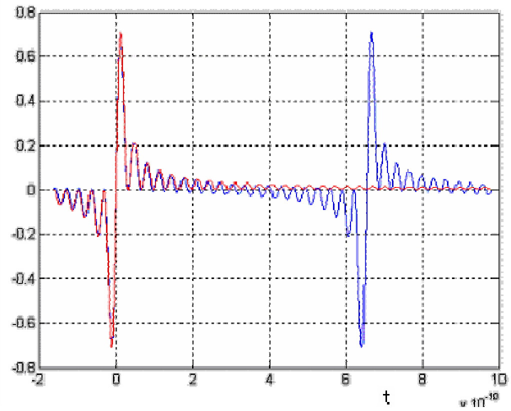


Figure 2 $\sum_{i=1}^{N}\sin(i\times 2\pi\times\Delta f\times t)$ and $\frac{1-\cos\omega t}{t}$

Thus analogy is elicited: $\cos\omega t \leftrightarrow \frac{\sin\omega t}{t}$ (4a); $\sin\omega t \leftrightarrow \frac{1-\cos\omega t}{t}$ (4b). Thus:

$$\cos[\omega(t+\tau)-\theta] \leftrightarrow \frac{\sin\omega(t-\tau)}{t}\cos\theta+\frac{1-\cos\omega(t-\tau)}{t}\sin\theta=\frac{\sin[\omega(t+\tau)-\theta]+\sin\theta}{t+\tau} \quad (5)$$

The author defines $\frac{\sin[\omega(t+\tau)-\theta]+\sin\theta}{t+\tau}$ sinuquotient, one extended trigonometric function wavelet, and $\frac{\sin\omega t}{t}$ and $\frac{1-\cos\omega t}{t}$ are defined sine sinuquotient and cosine sinuquotient respectively. The conventional trigonometric function wavelet has only one mother wavelet $\frac{\sin\omega t}{\pi t}$.

The following is the analysis of sine sinuquotient and cosine sinuquotient

1 whenω=0, $\frac{\sin\omega t}{t}\equiv 0$, $\frac{1-\cos\omega t}{t}\equiv 0$, so $\omega\neq 0$.

2 in the frequency domain, Fourier transform of $\frac{\sin\omega t}{t}$, F ($\frac{\sin\omega t}{t}$) is a gate function. We define a gate function as the following form:
G(x1,x2)=1, ∀ x∈ [x1, x2], G(x1,x2)=0 else x (6)
F ($\frac{\sin\omega t}{t}$)=πG(-ω,ω), F ($\frac{1-\cos\omega t}{t}$)=jπ[G(0, ω)-G(-ω,0)], $\omega\neq 0$

3 sinuquotient is belong to L^2, $\int_{-\infty}^{\infty}[\frac{\sin[\omega(t+\tau)-\theta]+\sin\theta}{t+\tau}]^2 dt<\infty$

4 sine sinuquotient and cosine sinuquotient have orthogonality.

$$\int_{-\infty}^{\infty}\frac{\sin\omega_1 t}{t}\times\frac{\sin\omega_2 t}{t}dt=\int_{-\infty}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}\pi G(-\omega_1,\omega_1)e^{j\omega t}d\omega\times\frac{\sin\omega_2 t}{t}dt=\pi\min(\omega_1,\omega_2) \quad (7)$$

From the above, $\frac{\sin\omega_1 t-\sin\omega_2 t}{t}$ is orthogonal with the $\frac{\sin\omega_1 t}{t}$ ($\omega_1>\omega_2$). And it is also proven that A=1 is reasonable. And cosine sinuquotient as well as sine sinuquotient has the same orthogonality.

5 sinuquotient and its translation

$$\int_{-\infty}^{\infty}\frac{\sin\omega_0 t}{t}\times\frac{\sin\omega_0(t+\tau)}{t+\tau}dt==\frac{\pi\sin\omega_0\tau}{\tau},\ \int_{-\infty}^{\infty}\frac{1-\cos\omega_0 t}{t}\times\frac{1-\cos\omega_0(t+\tau)}{t+\tau}dt=\frac{\pi\sin\omega_0\tau}{\tau}$$

$$\int_{-\infty}^{\infty}\frac{1-\cos\omega_0 t}{t}\times\frac{\sin\omega_0(t+\tau)}{t+\tau}dt=-\frac{\pi(1-\cos\omega\tau)}{\tau}\quad(8)$$

Let τ=0,the energy of sinuquotient is $\pi\omega$.

6 the relationship between sinuquotient and sinusoid.

Choose another function $\frac{\sin(\omega-\Delta)t}{t}$ to be subtracted from $\frac{\sin\omega t}{t}$:

$$\frac{\sin\omega t}{t}-\frac{\sin(\omega-\Delta)t}{t}=\frac{\sin\omega t-\sin(\omega-\Delta)t}{t}=\frac{2\cos(\omega-\frac{1}{2}\Delta)t\sin\frac{1}{2}\Delta\times t}{t}\quad(9)$$

when Δ->0, $\frac{\sin\omega t}{t}-\frac{\sin(\omega-\Delta)t}{t}\cong\Delta\sin\omega t$ ($\frac{1-\cos\omega t}{t}-\frac{1-\cos(\omega-\Delta)t}{t}\cong\Delta\sin\omega t$).

7 sinuquotient vs sampling function $\delta(t)$ and Hilbert transform.

Choose a function f(t) and its F(jω) has upper frequency f_h,and suppose $F_{\omega_0}(j\omega)$ is the component of F(jω) in $\omega\le\omega_0$ and $F_{\omega_0}^{-1}(j\omega)=f_{\omega_0}(t)$. Hilbert transform of $f_{\omega_0}(t)$ is $f_{\omega_0}^{h}(t)$, $F_{\omega_0}^{h}(j\omega)$ is the Fourier transform of $f_{\omega_0}^{h}(t)$:

$$\int_{-\infty}^{\infty}f(t)\frac{\sin(\omega_0 t)}{t}dt=\pi f_{\omega_0}(0)\ \ (12a).\ \int_{-\infty}^{\infty}f(t)\frac{1-\cos\omega t}{t}dt=\pi f_{\omega_0}^{h}(0)\ \ (10)$$

8 the energy feathers of sinuquotient

The table 1 gives the energy of sinuquotient in limited time (the number of sampling in time is 10000 in [0,2π/ω]). From the table, the sinuquotient can be truncated in [-5π/ω, 5π/ω] with little error.

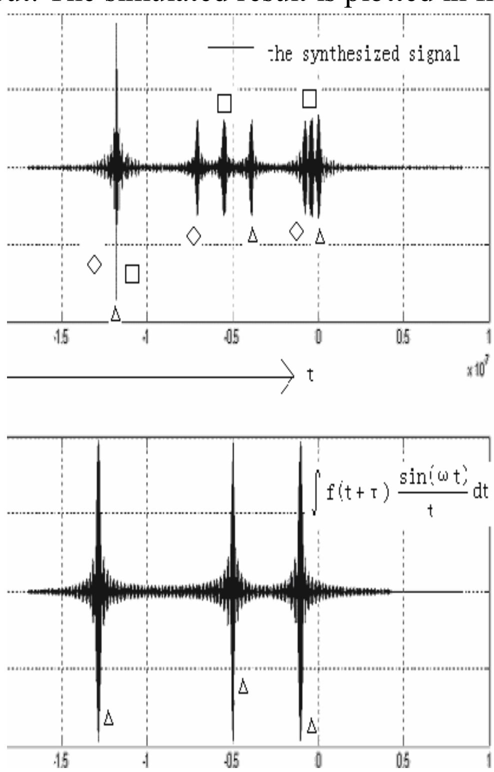
\Energy (100%)\t	[-π/ω, π/ω]	[-2π/ω, 2π/ω]	[-3π/ω, 3π/ω]	[-4π/ω, 4π/ω]	[-5π/ω, 5π/ω]
$\int[\frac{\sin\omega t}{t}]^2$dt	90.282%	94.994%	96.641%	97.475%	97.978%
$\int[\frac{1-\cos\omega t}{t}]^2$dt	65.262%	85.571%	89.578%	92.513%	93.854%

Table 1 the energy feathers of sinuquotient

Sinuquotient in UWB signal processing

Given there are 9 UWB signals (3 superposition), which have the same bandwidth 500MHz. There are 3 lines, center frequency 3.3GHz(Δtag), 3 lines, center frequency 4.0GHz(□tag), and 3 lines, center frequency 4.55GHz(□tag). The synthesized signal is plotted in the upper of figure 3 and 4. According to the sinuquotient orthogonality, if we choose a sinuquotient signal as the signal tagged by Δ to multiply the synthesized signal and integrate the product, the bandwidth-

500MHz-center frequency-3.3GHz component will be collected and others will be cut. The simulated result is plotted in figure 3 and 4.



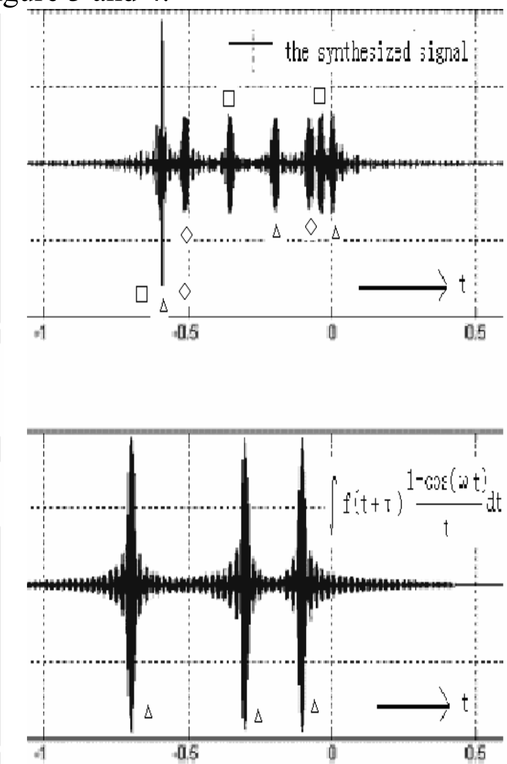


figure 3 done by sine sinuquotient

figure 4 done by cosine sinuquotient

Conclusion

The sinuquotient is a novel wavelet and it is simpler than other wavelets in frequency domain and has many perfect traits. And sinuquotient is a convenient tool to process the UWB signals. How to apply the sinuquotient to the practice is the further work in the future.

References:

[1] B. Natarajan, C.R.Nassar, S.Shattil "Throughput Enhancement in TDMA through Carrier Interferometry Pulse Shaping", VTC2000

[2] K.Pourvoyeur, A.Stelzer "Wavelet-based impulse reconstruction in UWB-radar", 2003 IEEE MTT-S Digest

[3] L. F. Chernogor and O.V. Lazorenko "Application of the wavelet analysis for detecting ultra-wideband signals in noise", MMET 2000 Proceedings