Network Concepts in Radiation and Scattering of Waves

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Introduction

Together with Nathan Marcuvitz Leopold Felsen has written the celebrated book "Radiation and Scattering of Waves" which is still an unsurpassed reference for systematically dealing with analytical electromagnetic field formulations [1]. In the preface of this book the authors stated: *"Concerning overall philosophy of subject presentation, much effort in this book has been expended on developing and applying a unified formalism for systematized eigenmode and transmission-line (network) analysis of linear field problems. ...... However, for analysis of classes of field problems having similar but not identical features, elimination of redundant aspects becomes almost essential."*

Partitioning of complex electromagnetic structures into subdomains joined together by interfaces, individual subdomains are characterized as circuit elements and the set of separation boundaries as the connection network. The compete electromagnetic structure then is represented by a generalized network formulation [2, 3, 4, 5]. Network-oriented methods can add significantly to the problem formulation and solution methodology and tan efficient parameterization of many electromagnetic problems.

Tellegen's Theorem and the Connection Network

Figure 1(a) shows the segmentation of a problem space into subregions R_i. The subregions R_i and R_j are separated by a boundary with the two sides B_{ij} and B_{ji}. The field formulation of Telegen's theorem is

$$\oint_{\partial V_B} \mathcal{E}'_{tr}(\boldsymbol{x}, t') \wedge \mathcal{H}''_{tr}(\boldsymbol{x}, t'') = 0\,, \tag{1}$$

where $\mathcal{E}'_{tr}(\boldsymbol{x}, t')$ and $\mathcal{H}''_{tr}(\boldsymbol{x}, t'')$ represent the electric and magnetic field differential forms [4, 5]. The integration has to be performed over both sides of all boundary surfaces. Prime ' and double prime " denote that in both cases time and also material filling of the subregieons may be different. The equivalent circuit representation is depicted in Figure 1(b). The network representation of Tellegen's theorem is

$$\sum_{n}^{N_\alpha} V_n^{\alpha'}(t') I_n^{\alpha''}(t'') + \sum_{n}^{N_\beta} V_n^{\beta'}(t') I_n^{\beta''}(t'') = 0\,, \tag{2}$$

where the generalized voltages and currents $V_n^{\alpha'}(t')$ and $I_n^{\alpha'}(t')$ are the expansion coefficients of the tangential components of the electric and magnetic fields at the boundary surfaces.

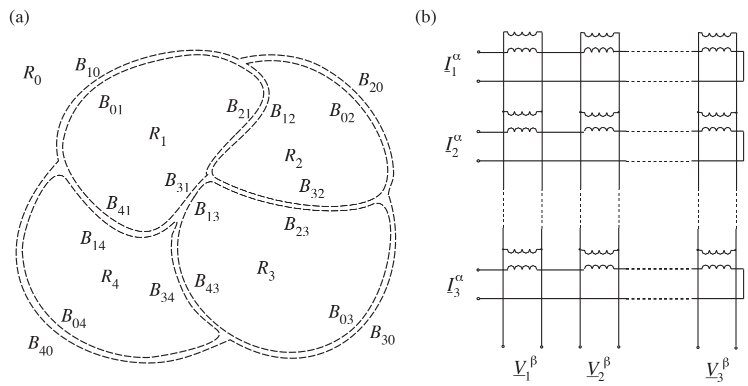

Figure 1: (a) Segmentation of an electromagnetic structure and (b) canonical form of the connection network.

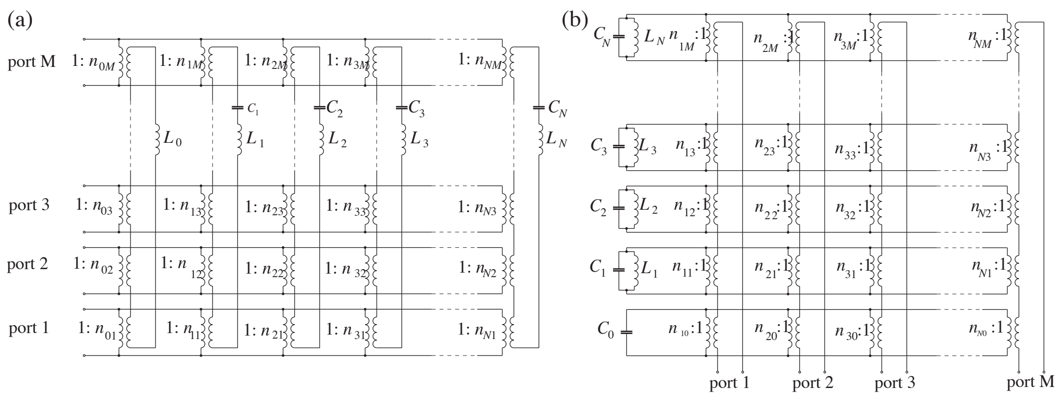

Figure 2: Foster (a) admittance and (b) impedance representations of a multiport.

The Characterization of Circuits and Subcircuits

We may write the transverse field expansions and the field solutions may be expressed in integral form [4, 5] in terms of the electric dyadic impedance and admittance Green's forms respectively [6, 4, 5]. Expanding the electric and magnetic field forms $\underline{\mathcal{E}}$ and $\underline{\mathcal{H}}$ into a series of biorthonormal structure forms we obtain generalized voltages an currents, with immittance matrix elements that can be represented by multiport Foster impedance and admittance representations as shown in Figure 2 [4, 5].

Spherical Modes and Free Space

By considering wave propagation along a radial waveguide in the spherical coordinate system, a complete description of free space is possible in terms of a discrete (but infinite) number of radial transmission lines [7, 1, 8].

The complete electromagnetic field o may be expanded into a set of TM and TE spherical waves [9]. Using the recurrence formula for spherical Bessel functions, Chu gave the Cauer representationof the equivalent circuits of the TM_n and the TE_n

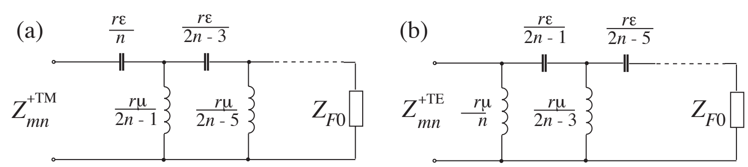

Figure 3: Equivalent circuit of (a) TM_{mn}, (b) TE_{mn} spherical wave.

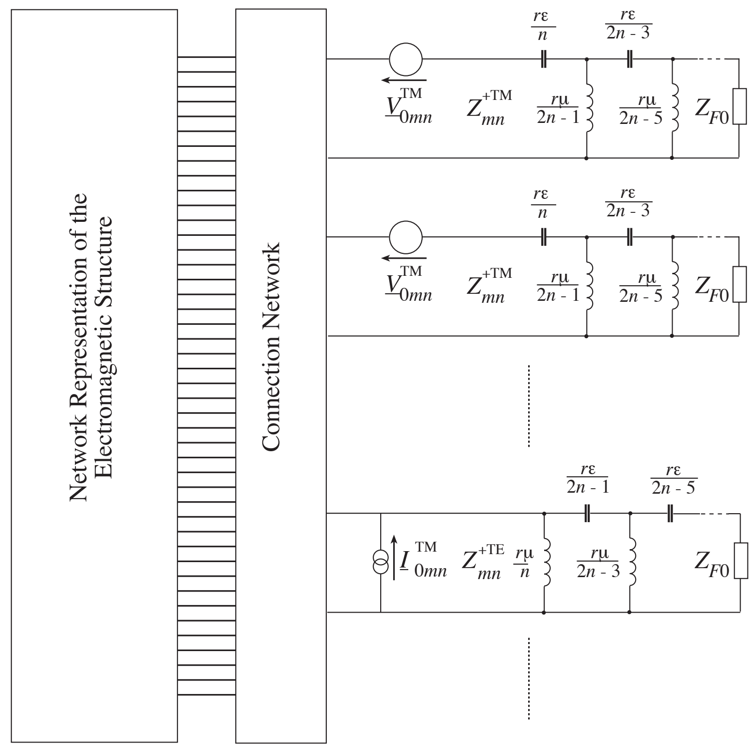

Figure 4: Embedding of an electromagnetic structure into a sphere.

spherical waves. The spherical TM_{mn} and TE_{mn} fields can be derived from scalar potentials [9, 1] in terms of the associated Legendre polynomials and of the spherical Hankel functions describing the outward propagating waves. The continued fraction expansions of wave impedances for the outward propagating TM and TE modes represent the *Cauer canonic realizations* shown in Figure 3(a), (Figure 3(b). We note that the equivalent circuit representing the TE_{mn} mode is dual to the equivalent circuit representing the TM_{mn} mode. The equivalent circuits for the radiation modes exhibit highpass character. For very low frequencies the wave impedance of the TM_{mn} mode is represented by a capacitor $C_{0n} = \varepsilon r/n$ and the wave impedance of the TE_{mn} mode is represented by an inductor $L_{0n} = \mu r/n$. For $f \to \infty$ we obtain $Z_{mn}^{+\text{TM}}$, $Z_{mn}^{+\text{TE}} \to Z_{F0}$.

Network Description of the Scattering of Waves

For a any finite frequency interval we can specify equivalent circuits with a finite number of lumped elements that models the properties of the electromagnetic structure with arbitrary acurracy. Figure 4 shows the equivalent circuit of an electromagnetic structure. Let the structure be embedded in a virtual sphere with radius r. The network representation of a structure consisting of linear reciprocal lossless material can be given by a Foster equivalent circuit according to Figure 2. Via a connection circuit as specified in Figure 1 the the electromagnetic structure is coupled to the radiation modes. The equivalent circuit contains a one-port equivalent circuit for every considered TM_{mn} and TE_{mn} radiation mode respectively. Following [10] an incident plane is represented by a voltage source $\underline{V}_{0mn}^{+\mathrm{TM}}$ for every TM_{mn} radiation mode and a current source $\underline{I}_{0mn}^{+\mathrm{TM}}$ for every TE_{mn} radiation mode as shown in Figure 4.

Conclusions

The pioneering work of Professor Felsen [1] has paved the way for the *systematic* solution of field problems. Together with the connection network it allows to translate field problems into network problems. In particular the connection network, which maps different field representations, allows to write the generalized transformer of the delta representation, i.e. the network equivalent of the completeness relationship. Spherical coordinates and in particular radial transmission lines and angular eigenfunction expansions provide a suitable means for the systematic translation of field problems in open space into network equivalents.

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