Strong Relations between the Variances of Multi-Port Parameters in Stochastic Environments

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In this contribution, we first show how the model parameters (scattering, impedance or admittance, ...) of a passive multi-port (electronic) system are affected by the scattering properties of obstacles in the system's environment. Then, we prove, in the context of macroscopic, time-harmonic, electromagnetic theory, that if this environment is a stochastic environment with a statistically isotropic scattering operator, the induced fluctuations in the multi-port parameters show the so-called Hauser-Feshbach property. This phenomenon has first been observed experimentally in the context of S-parameter measurements in Mode-Stirred Chambers. Subsequent work has suggested that such relations are "universal" which has inspired the theoretical development presented in this contribution (see also Fiachetti and Michielsen, El. Letters Vol.39 No.24).

The fact that the parameters of a multi-port system are influenced by the system's environment is a simple consequence of the back-scattering of the fields emitted by the system when sources are applied to its ports. Starting point of our analysis is a rigorous relation between the environment scattering operator and the induced variations, with respect to a reference environment, in the multi-port model parameters.

If the environment has uncertainties the multi-port model parameters will have uncertainties too. These uncertainties may be due to fluctuations in time or fluctuations in the realisation (in the statistical sense) of the environment. For example, the environment of the system may change in unpredictable ways, as a function of time, but the environment may also be fixed in time having only uncertainties as to its true geometry or physical constitution.

In order to account for these uncertainties we model the environment electromagnetically by means of a stochastic scattering operator. The back-reaction of the environment on the system, implies that the multi-port model parameters are to be considered as stochastic variables (or as having a stochastic perturbation around their intended deterministic values). We shall prove that if a lossless stochastic environment's scattering operator is statistically isotropic (in a way we shall make precise) the fluctuations in the multi-port model parameters are related in a very special way. The essential relations we obtain appear to be the same as those already identified by Hauser and Feshbach in their study of neutron scattering in 1952.

There is a second approach to this kind of relations, which is being developed at the University of Maryland (see X. Zheng, S. Hemmady, T. Antonsen, S. Anlage and E. Ott, arXiv:cond-mat/0504196). This approach is based on the analysis of environments with a specific, given, geometry which shows so-called "wave-chaos." The statistics in the multi-port parameters in this case correspond to uncertain position of the system in such an environment. This leads to exactly the same relations between the fluctuations, however, the Maryland approach has also been applied to environments with losses. It is our purpose to show that these generalisations can also be obtained by extending the above established scattering operator based theory to lossy environments.