Linear Source in a Circular Tunnel

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We develop a solution for the fields in a circular tunnel with a lossy wall, excited by a linear current source. The problem has application to wireless communication in both caves and tunnels. The case of ϕ-independent ring source excitation has been given recently (D. G. Dudley, *IEEE Trans. Antennas Propagat.*, 53, 435-441, 2005) in a development appropriate for producing the ϕ-symmetric fields, TE_{0m} and TM_{0m}. Herein, we consider in detail the fields produced by a linear current source at an arbitrary location in the tunnel. Such a source produces a much richer family of modes, including the above plus the hybrid modes, HE_{nm} and EH_{nm}.

We formulate the problem in cylindrical coordinates. We take transforms over both the axial and angular variables and then solve ordinary differential equations over the radial coordinate. We produce the homogeneous solution in the transform domain in both the tunnel and the surrounding medium. We add to the solution in the tunnel a particular solution produced by a linear source radiating in free space. We apply boundary conditions resulting in a matrix equation whose inversion yields coefficients that complete the expressions for the fields. The homogeneous form of the matrix equation is the characteristic equation for the modes. We use FindRoot in Mathematica to solve for the roots of the equation. We discuss initial estimates for the roots and investigate in detail the final root positions. Inverse transforms yield a double series over the angular variable and the residues at the roots. We include an in-depth description of how the series are summed and how we check for convergence. We include representative plots for the fields as a function of axial distance.

The results in this paper provide a model with which to study channel capacity in wireless tunnel communication. We comment on the possibility of exploiting the Foschini-Telatar channel capacity model. These ideas have been discussed (M. Liénard, et. al., *IEEE J. on Selected Areas in Comm.*, 21, 332-339, 2003) in conjunction with measurements in the Paris subway system.