Time-Reversal MUSIC Imaging of Extended Scatterers, and a Non-Iterative Exact Inverse Scattering Formula

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Time-reversal MUSIC imaging of spatially extended scatterers has been investigated for scalar fields in the linearizing framework of the Born approximation in (Zhao, SIAM J. Appl. Math., 64, p. 725-745, 2004) and (Hou et al., J. Comp. Phys., 199, p. 317-338, 2004). The forward aspects of time-reversal of extended targets have been investigated for electromagnetic fields in, e.g., (Chambers and Berryman, IEEE Trans. Antennas Prop., 52, p. 1729-1738, 2004). In this talk a general theory is presented for a time-reversal MUSIC imaging of penetrable extended scatterers (characterized by a scattering potential function) using scalar and electromagnetic fields. The scatterers are assumed to be embedded in a known but rather general linear and reciprocal background medium. The developments hold within exact scattering theory; thus no linearizing approximations are involved in the associated forward model, and the imaging or support inversion of the scatterers holds not only for the special case of sub-wavelength or point targets which has been the focus of time-reversal presentations but also for the more general case of objects of rather arbitrary shape and dimensions including objects having simply or not simply connected support regions such as collections of targets plus clutter in disjoint domains. Importantly, the theory is developed for both time-harmonic and broadband probing fields. While for a point target there is a single Green function vector or propagator per target, for an extended target one must associate a countably infinite number of communicable modes or propagators per target. For a given SNR the space of such communicable modes becomes essentially a finite-dimensional one, and the imaging function or MUSIC pseudo-spectrum $P(D_o')$, where D_o' represents a given support, with D_o being the actual scatterer support, reads (in receive) as follows: $P(D_o') = \left[\sum_{p=N(D_o)+1}^{N_{\text{tot.}}} \sum_{q=1}^{N(D_o')\leq N(D_o)<N_{\text{tot.}}} |(\Psi_p, \psi_{r,q}^{(D_o')})_{Y_{\text{eff.}}}|^2\right]^{-1}$ where: $N_{\text{tot.}}$ is an essential number of degrees of freedom; (Ψ_p, Q_p, σ_p) is the singular system of the scattering or data matrix K as measured with the given remote sensing system, so that $K = \sum_p \sigma_p |\Psi_p >< Q_p|$ in Dirac bra-ket notation, and for which $p > N(D_o)$ represents negligible singular values σ_p; $(\psi_{r,q}^{(D_o')}, J_q^{(D_o')}, \lambda_q^{(D_o')})$ is the singular system of the radiation mapping from the sources induced within the scatterer support D_o' to the measured scattered fields at the receiver system, and for which $q > N(D_o')$ represents negligible singular values $\lambda_q^{(D_o')}$; and $(\cdot,\cdot)_{Y_{\text{eff.}}}$ represents the non-weighted L^2 inner product for L^2 fields $\in Y_{\text{eff.}}$ measured within the receiver support. If $D_o' \subseteq D_o$ then $P(D_o')$ peaks which provides a time-reversal MUSIC image of the objects. Finally, a new single-step, non-iterative formula for the reconstruction of not only the unknown scatterer support (as done via the time-reversal MUSIC approach in the previous step) but also the unknown scattering potential or index of refraction is presented that is applicable to collections of point scatterers or to certain scatterers whose forward response for the given remote sensing system can be suitably modeled using a computational grid corresponding to a finite number of dominant scattering centers. Remarkably, this non-iterative formula holds within exact scattering theory despite the nonlinearity of the scattering potential-to-data mapping.