A Hybrid Spectral Integral Method / Spectral Element Method for Electromagnetic Wave Scattering

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Spectral integral methods (SIMs) have been studied by many researchers in the last decade due to their high accuracy. Atkinson and Ganesh et. al. have studied the acoustic wave scattering from smooth objects by using SIM in three-dimensions (3D) (K. E. Atkinson. SIAM J. Numer. Anal., 19:263–274, 1982; M. Ganesh and I. G. Graham. J. Comp. Phys., 198:211–242, 2004), and Liu et. al. have used SIM to solve electromagnetic wave scattering problems in two-dimensions (2D). But 3D SIM for electromagnetic wave scattering still has not been developed.

Recently, Liu *et al.* proposed that the spectral integral method can be utilized as an efficient radiation boundary to truncate the computational domain in the finite element method in a homogeneous background medium (J. Liu and Q. H. Liu, submitted for publication) and for layered media (E. Simsek, J. Liu, and Q. H. Liu, submitted for publication) in two dimensions. It has been shown that the required number of points on the radiation boundary is only around 5 points per wavelength, thus giving substantial saving in memory and CPU time requirements compared with the method of moments.

Similar to this approach, here we propose a hybrid spectral-integral/spectral-element method (SIM/SEM) to solve three-dimensional electromagnetic and acoustic wave scattering problems. Lee et. al. have developed a spectral element method (SEM) to obtain spectral accuracy with the orders of the basis functions. The SEM can be considered as a special class of the general finite element method (FEM), with the choice of nodal points and quadrature integration points based on Gauss-Lobatto-Legendre (GLL) polynomials.

In the proposed hybrid SIM/SEM technique, an artificial smooth boundary outside an inhomogeneous and arbitrarily shaped object is used as a radiation boundary, where the SIM is applied to solve the integral equation for the exterior region; the SEM is applied to the interior region, including the boundary, to solve the differential equation. The use of Fourier series bases makes it possible to accurately interpolate between SEM nodal points and SIM nodal points. The combined SIM/SEM provides spectral accuracy for both the interior differential equation method and the boundary integral method. Numerical examples will be shown to demonstrate this spectral property.