

Discrete Wavelet Transform Compression for Time Domain Integral Equations

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In recent years, various fast algorithms for accelerating the solution of time domain integral equations (TDIEs) have been proposed. Multilevel space-time FFT-based algorithms for accelerating TDIE solution schemes are particularly promising because they apply not only to free-space problems, but also, with minimal modifications, to layered, lossy, and dispersive media problems without significant increase in computational complexity. Whenever the Green's function has a temporal tail, the classical marching-on-in-time (MOT) analysis of a scattering problem requires $O(N_t N_s^2)$ memory and $O(N_t^2 N_s^2)$ operations for N_t simulation time steps and N_s spatial samples. FFT-based algorithms, on the other hand, exploit the space-time translational invariance of the integral kernel and reduce this memory requirement to $O(N_t N_c)$ and the computational complexity to $O(N_t N_c \log(N_t N_c) \log N_t)$. Here N_c is either N_s for uniform scatterer discretizations, or the numbers of nodes on an auxiliary projection mesh for nonuniform discretizations.

In practice, when the scatterer grows electrically large, the bottleneck of FFT-accelerated TDIE solvers is their storage requirement. While the overall memory requirement is better than that of classical MOT solvers (N_c grows as at most $N_s^{3/2}$), the dependence on N_t limits the duration of the analysis, thereby constraining the applicability of these methods to nonresonant scenarios. Here, we report on a discrete wavelet transform-based compression scheme that reduces the memory appetite of both classical MOT solvers and their FFT-based accelerators. In contrast to previous efforts to reduce the memory requirements of TDIE solvers—in which wavelets were used to compress either the MOT spatial memory demands or the spatio-temporal memory demands of global solvers—here, *wavelets are used to efficiently compress temporal waveforms within an MOT framework*. Such compression is possible because, for example, typical MOT solvers require the current waveforms to be heavily oversampled to obtain accurate solutions. However, this high sampling rate is not required to store past current values; in the end, they are represented by bandlimited waveforms. Specifically, while $O(N_t N_c)$ distinct values are required by the FFT-accelerated TDIE solvers (to store the impedance matrices, current history, and partially computed future field values), these values are *not* needed at all N_c space- and N_t time-points *simultaneously*. For example, at each spatial point the known current values can be uncompressed right before they are needed and compressed right after they are used. If this procedure were repeated at each time step, however, its complexity would be of $O(N_c N_t^2)$. Rather, a multilevel compression scheme is used that in structure matches the treatment of temporal FFTs in FFT-accelerated solvers: progressively larger blocks of currents are compressed at larger time-steps, and the larger the block the less often it is compressed. This leads to a complexity of $O(N_c N_t \log N_t)$ for the compression scheme.