Efficient Solution of Time Domain Volume Integral Equations Using the Adaptive Integral Method

Ali E. Yılmaz[1]*, Balasubramaniam Shanker[2], Jian-Ming Jin[1], and Eric Michielssen[1]

[1] Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

[2] 2120 Engineering Building, Department of Electrical and Computer Engineering, Michigan State University, East Lansing, MI 48824, USA

While time domain surface integral equation formulations typically lead to the most efficient methods for analyzing scattering from nonpenetrable or homogenous penetrable objects, their volume extensions are required when modeling inhomogenous scatterers (N. T. Gres et al., *Radio Sci*. **36(**3), 379-386, 2001). When compared to differential equation solvers, such as those based on the finite-difference time-domain (FDTD) method, volume integral equation solvers inherit most advantages of their surface counterparts. In particular, they (i) implicitly impose the radiation condition, (ii) do not discretize space around the scatterer, (iii) do not suffer from phase dispersion errors, (iv) do not constrain the time-step size by the spatial discretization dimensions (K. Aygün et al., *Int. J. Num. Mod.: Elect. Net. Dev. & Fields* **15,** 439-457, 2002), and (v) because of all of the above, typically require fewer unknowns for a given accuracy. Despite all these features, the high computational complexity and memory requirements of classical marching-on-in-time algorithm (MOT) based integral equation solvers render them unpopular compared to differential equation solvers. Indeed, classical MOT computational complexity and storage requirements scale as $O(N_t N_v^2)$ and $O(N_v^2)$, whereas FDTD simulations typically require $O(N_t' N_v')$ operations and $O(N_v')$ storage. Here N_t and N_v are the number of MOT time steps and spatial vector basis functions (typically the volumetric rooftop basis functions), and N_t' and N_v' are the number of FDTD time steps and grid points such that both solvers simulate fields throughout the scatterer for the same duration and with similar accuracies.

Recently, the time domain counterpart of the adaptive integral method (TD-AIM) for accelerating the solution of time domain surface integral equations has been developed. TD-AIM pre-corrects near-field interactions and efficiently computes far-field interactions through multilevel space-time fast Fourier transforms. The fast Fourier transforms are facilitated by an auxiliary (volumetric) uniform mesh that contains the scatterer (unlike the FDTD grid, this mesh does not extend beyond the scatterer since there is no explicit termination by absorbing boundary conditions). When applied to quasi-planar scatterers the TD-AIM algorithm achieves linear complexity (within a logarithmic factor) for surface integral equations. In this work, TD-AIM is extended to volume integral equations, where it also achieves near-linear complexity. Specifically, the computational complexity and memory requirements of TD-AIM for the solution of volume integral equations are $O(N_t N_c \log(N_g N_c) \log N_g)$ and $O(N_g N_c)$, respectively, where N_c is the number of nodes on the uniform mesh and N_g is the maximum transit time in time steps across the scatterer (i.e., the length of the current history that has to be stored). For volume integral equations, $N_g \sim N_v^{1/3}$ and $N_c \sim N_v$.