Stability of the Time Domain Finite Element Method

Thomas Rylander* and Jian-Ming Jin
Center for Computational Electromagnetics
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801-2991

The finite element method (FEM) formulated in the time domain is attractive for devices with complex geometry in three dimensions where an accurate result is required in a wide frequency band. For time domain computations in general, it is well known that a numerically stable time-stepping scheme is necessary for reliable results. This talk features stability issues related to the time domain FEM.

The talk will include a review of some recent results on the stability of explicit-implicit hybrid time-stepping schemes for Maxwell's equations (Thomas Rylander and Anders Bondeson, "Stability of explicit-implicit hybrid time-stepping schemes for Maxwell's equations," *J. of Comput. Phys.*, vol. 179, no. 2, pp. 426-438, July, 2002). The recently developed and new hybrid scheme combines the efficiency of the finite-difference time-domain (FDTD) scheme with the ability of the FEM to model complex boundaries. In fact, the FDTD scheme can be deduced by FEM techniques, i.e. edge elements on cubes with lumping by trapezoidal integration, and consequently the hybrid method can be considered as a pure FEM. The hybrid method is derived by applying the Galerkin's method to the self-adjoint Maxwell's equation. This implies that the discretized "$\nabla \times \mu^{-1} \nabla \times$"$-$ and "ϵ"$-$operators are real symmetric semi-positive definite and positive definite matrices, respectively. The eigenvalues k^2 of the corresponding eigenvalue problem are therefore real and non-negative which is necessary for stability. In fact, the hybrid algorithm is stable for time steps up to the stability limit of the FDTD without added dissipation and the proof of stability will be presented during the talk. The hybrid scheme is free from spurious solutions.

We are currently working toward a generalization of this proof of stability, which is intended to include the recently developed conformal perfectly matched layers for the time domain FEM (Thomas Rylander and Jian-Ming Jin, "Conformal Perfectly Matched Layers for the Finite Element Method in the Time Domain," submitted to *2003 IEEE AP-S International Symposium*). The status of this work will be reported during the talk.