Computing the Translation Operator by Sampling the Green Function at Low Frequencies

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Efficient numerical simulation of integral formulations of electromagnetic problems entails the use of a compression algorithm for the underlying Green function. If a simulation domain is subdivided into N elements, a traditional discretization of the integral equation leads to a linear system of the form $Ax = b$ where A contains $O(N^2)$ nonzero elements. Several methods have been developed to reduce this cost scaling to $O(N \log N)$ or less. An important group of these techniques are the Fast Multipole Methods (FMMs). The earliest form of the FMM algorithm (Greengard and Rokhlin, *J. Comp. Phys.*, 1985) is applicable to low-frequency problems. A more recent FMM algorithm, based on a diagonalization of the translation operator, is efficient at higher frequencies (Rokhlin, *J. Comp. Phys.*, 1990).

An important feature of these FMMs is their dependence on a detailed analysis of the underlying Green function. The analysis facilitates the numerical implementation of a compressed representation of the free-space kernel without sampling the underlying, dense operator. Unfortunately, this feature of the FMM also makes it difficult to directly extend the method to Green functions which satisfy more general boundary conditions.

As discussed in separate papers presented at this meeting, a technique has been developed which determines a compressed, plane wave representation of the Green function at high frequencies using only sparse numerical samples of the Green function. In this presentation we will discuss an analogous procedure for compressing the Green function at low frequencies. As in the high-frequency algorithm discussed elsewhere, the low-frequency algorithm discussed here is based on an asymptotic expansion of the Green function. The low frequency translation operator is subsequently determined from sparse numerical samples of the underlying Green function.

Asymptotic matching of the high-frequency and low-frequency expansions will be shown to yield a uniform compression algorithm for the free-space Green function based only on asymptotic representations satisfying the wave equation. The extension of these results to Green functions satisfying more general boundary conditions is discussed in a separate presentation.