Error Optimization in Fast Multipole Algorithm

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Abstract

Electromagnetic wave scattering problems involving electrically large complex objects, for example, aircraft, rocket, automobile and spacecraft have been intensively investigated with the advancement of the computer. In order to solve large complex objects using method of moments (MoM), a surface is discretized into finite elements and the integral equations with the unknowns on each finite element are formulated. The resultant matrix equation comprises all interactions between field and source points is often solved by iterative method like the Conjugate Gradient (CG) method-it takes an extremely long time if especially large scattering objects with a large number of unknowns are solved.

The fast multipole algorithm (FMA) and multilevel fast multipole algorithm (MLFMA) are methods to remedy the unprecedented time-consuming calculation by diagonalizing the matrix elements expressed by interactions between the field and the source points with the assistance of the addition theorem in MoM. In this algorithm, the Green's operator is factorized and expressed in more than three steps; aggregation β_{li}, diagonal translation $\alpha_{l'l}$ and disaggregation $\beta_{jl'}$ processes.

$$H_0^{(1)}\left(k\rho_{ji}\right)=\frac{1}{2\pi}\int_0^{2\pi} d\alpha\tilde{\beta}_{jl'}(\alpha)\tilde{\alpha}_{l'l}(\alpha)\tilde{\beta}_{li}(\alpha) \tag{1}$$

where

$$\tilde{\alpha}_{l'l}(\alpha)=\sum_{p=-P}^{P} H_p^{(1)}\left(k\rho_{l'l}\right)e^{-ip\left(\phi_{l'l}-\alpha+\frac{\pi}{2}\right)} \tag{2}$$

and

$$\tilde{\beta}_{jl'}(\alpha)=e^{-ik\rho_{jl'}\cos\left(\alpha-\phi_{jl'}\right)} \qquad \tilde{\beta}_{li}(\alpha)=e^{-ik\rho_{li}\cos\left(\alpha-\phi_{li}\right)} \tag{3}$$

Since the Hankel functions have the tendency of divergence, the truncation number P of the diagonal translation operator $\alpha_{l'l}$ must be determined carefully in the numerical implementation. In order to calculate faster with high precision, the optimum truncation number P should be determined as small as possible. The truncation number P is conventionally determined by the number of digits of accuracy d_0.

Ohnuki and Chew have proposed the novel expression of the number of digits of accuracy that is defined by d_0-d_1. The d_0 part stands for the convergence rate of the Bessel functions in the multipole expansion. Therefore, this value is determined by the element location. For conventional use, the maximum box size equals to the diagonal length of the box is chosen to predict the error bound. On the other hand, the d_1 part stands for the divergence rate of the Hankel function of the diagonal translation operator. This is determined by the distance between the two box centers.

This paper clarifies the relation between the various parameters, for instance, truncation number, buffer number, box number, element location, element number. The novel truncation number P is determined as the function of not only the element location but also the buffer number. The novel truncation number defined by the novel rule is smaller than the conventional truncation number. Time reduction by adopting the novel rules together with the excellent calculation precision is expected.