

## **Multi-Level Fast-Multipole Analysis of Induction Problems**

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The sensing of buried unexploded ordnance is typically performed with magnetometers and via electromagnetic induction (EMI). Magnetometers are passive, sensing the presence of a subsurface ferrous target by detecting subtle local changes in the earth's magnetic field. EMI represents an active sensor, in which low-frequency fields are emitted and the scattered fields measured. To mitigate losses in the soil, EMI sensors typically operate at kilohertz frequencies. If a buried target is non-ferrous, as are many buried unexploded ordnance, for example, then EMI is required (such targets are invisible to a magnetometer).

It is of significant importance to understand the EMI signature of buried metal targets, since such a signature is ultimately used for inversion (to determine whether the subsurface target is a UXO, or clutter). In this paper we consider the numerical analysis of EMI sensing of general conducting and ferrous targets. A frequency-domain volumetric integral equation formulation is employed. Since EMI frequencies are considered, the target may not be modeled as a perfect electric conductor (e.g. the properties of the metal play an important role in the target's EMI signature).

Although the UXO are typically very small with respect to the EMI wavelength, the numerical analysis is very expensive computationally. In particular the size of the volumetric basis functions must be small relative to a skin depth. Since the skin depth is typically very small relative to the size of the target, the number of unknowns  $N$  may become quite large, significantly taxing memory and computational resources. We therefore analyze the volumetric integral equation by an extension of the fast-multipole method to low frequencies.

The numerical formulation is discussed in detail, and the results of the model are compared to measurements.