Fast Multipole Algorithm Frame Analysis

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FMA (Fast Multipole Algorithm) is becoming popular with the increasing computer capabilities and complex electromagnetics requirements. Its principles, cost and memory distributions have an important impact on the efficiency of the applications based on it. A careful FMA frame analysis can guide researchers to develop an optimized algorithm. This study is meant to analyze how FMA distributes the computational loads, and find the ways to improve the FMA performance.

The most popular FMAs focus on the static cases and high-frequency dynamic cases. In principle, they are all based on the addition theorem, albeit with different implementations. The far-field interactions are computed through aggregating the outgoing waves, translating the outgoing waves into the incoming waves, and disaggregating the incoming waves. The near-field interactions are evaluated directly. The speedup of FMA over MoM comes from the off-diagonal matrix-vector product approximated by error-controllable factorized far-field interactions. At least one buffer box is necessary to make the error controllable. So there are MoM (near field) and FMA (far field) parts in a complete FMA procedure. The cost balance between them is important and needs to be optimized.

Since the static Laplacian equation is scale invariant, the multipole expansion of the static Green's function can be scaled so that all levels share the same translators. However, these translators are dense matrices. Then most computational time is spent on the outgoing-wave to incoming-wave translation stage. Of all levels, the finest level has the heaviest load. As a result, the total cost of the static FMA is $O(N)$ (where N is the basis function number).

For the dynamic case, the wave equation is scale variant. In contrast, the dynamic translators are diagonal and cannot be shared between levels. Since the radiation and receiving patterns only compose of propagating waves, the dynamic FMA suffers from low-frequency breakdown. Because of the oscillatory field, the radiation patterns of different levels in the dynamic FMA have different bandwidth. So interpolations and anterpolations are needed in the aggregations and disaggregations to reduce the workload. Analysis shows that when the level number is large, the computational cost of aggregation and disaggregation will become very large. The load of every level is of the same order, which means that the FMA frame shifts the cost to different levels to improve the general performance. As a result, the total dynamic cost will be $O(N \log N)$.

Cost estimation equations and simulation data will be presented in the conference to support the above statements derived from the FMA frame analysis. Suggestions on how to improve the efficiency of FMA will be provided.