Numerical Evaluation of the Polarizabilities of Complex Scatterers with High-order Boundary Element Model

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When a dielectric inclusion is put into a homogeneous electric field, it causes a perturbation to the total electric field distribution. The perturbation is concentrated in the neighborhood of the inclusion. The dipole moment induced in such a scatterer can be identified with the source of the dipolar component in the scattered field, and it is proportional to the external electric field. The ratio between these two, the induced dipole moment and the external field is called polarizability of the scatterer and it is an important parameter in electrostatics, quasistatics, and also in electrodynamics and in modelling of materials.

The polarizability can be calculated by solving boundary integral equation

$$\phi_e(\mathbf{r}) = \frac{\tau+1}{2}\phi(\mathbf{r}) + \frac{\tau-1}{4\pi}\int\limits_S \phi(\mathbf{r}')\frac{\partial}{\partial n'}\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) dS', \quad \mathbf{r} \text{ on } S \tag{1}$$

for the unknown potential function ϕ. In this equation, S is the surface of the inclusion, $\phi_e = -E_e z$ is the potential of the incident field, and ϕ is the total potential on the surface. The ratio of the permittivities is denoted by $\tau = \epsilon_i/\epsilon_e$, where ϵ_i is permittivity of the inclusion. The outward normal to the surface is $\mathbf{n}'$ at point $\mathbf{r}'$.

Equation (1) is solved in this work by the Galerkin method. The surface is discretized with planar triangles, and the potential and the test functions are spanned with third order piecewise polynomial basis functions. In order to ensure accuracy of the solution, the inner integrals of those terms where $\mathbf{r}$ and $\mathbf{r}'$ are close to each other are evaluated analytically.

In the talk, results will be shown for the polarizability of dielectric polyhedra (cube, tetrahedron, octahedron, dodecahedron, icosahedron) and also for the response of clusters of two spheres which are used for the dielectric modelling of dry snow.