Fast Global Radiation Boundary Conditions Based on Non-uniform Grid Approach

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Differential equations based techniques, such as finite element and finite difference methods, are often preferred for analyzing electromagnetic problems including inhomogeneous materials. For configurations defined in unbounded domains, these techniques must be augmented by absorbing boundary conditions, which allow for the truncation of the computational domain. Conventional local Absorbing Boundary Conditions (ABCs) including the Perfectly Matched Layer (PML) type absorbers facilitate such mesh truncation as long as the boundary surface is convex. The convexity requirement translates into a significant additional computational cost when treating an essentially concave geometry since a relatively large "white space" must be meshed in order to obtain a convex exterior boundary. Alternatively, boundary integral formulations allow for arbitrary shaped exterior boundaries, however at the cost of making the boundary conditions global. Therefore, in order to be computationally competitive, the boundary integral operators must be evaluated invoking fast algorithms.

In this paper we propose the use of the recently proposed Non-uniform Grid (NG) approach (Boag et al., *IEEE Antennas Wireless Propagat. Lett.*, vol. 1, no. 7, pp. 142-145 2002.) for a fast evaluation of the boundary integrals. As an example we consider a two-dimensional scattering by a thin concave shell forming a large open-ended cavity. In addition, we assume the usage of a generic iterative solver that becomes a necessity for analysis of electrically large problems. With a conventional ABC approach requiring a convex exterior boundary, the entire interior of the cavity is meshed resulting in a number of unknowns proportional to the electrical dimensions of the scatterer squared, i.e., of $O(N^2)$, where $N = kR$, R being the radius of the smallest circle circumscribing the scatterer and k is the wavenumber. A conventional approach would require $O(N^2)$ operations at each iteration step. On the other hand, if the computational domain is confined by a conformal boundary, the number of unknowns and boundary points is proportional to the electrical dimensions of the scatterer, i.e., of $O(N)$. Clearly, direct evaluation of the boundary integral would then require $O(N^2)$ operations, i.e., the same order as the ABC based approach. This high computational burden underlines the need for using fast field evaluation techniques.

We will demonstrate that the usage of the two-level NG algorithm reduces the computational cost of evaluating the boundary integrals (a single iteration) from $O(N^2)$ to $O(N^{3/2})$. The multilevel algorithm attains an asymptotic complexity of $O(N \log N)$.