3D Modeling of Geoelectromagnetic Fields Using a Fast Integral Equation Approach

Dmitry Avdeev *, Alexei Kuvshinov , Oleg Pankratov
Russian Academy of Sciences, Geoelectromagnetic Research Institute, PO Box 30, 142190 Troitsk, Russia, e-mail: d.avdeev@mtu-net.ru

1. Introduction.

We have developed a new series of 3D forward modeling codes for various geoelectromagnetics (geoEM) applications, including: (1) induction logging in deviated wells, (2) grounded and airborne controlled-source EM, (3) magnetotellurics (MT), and (4) global induction studies.

This series of codes has proven to be effective way to simulate geoEM fields in complex 3D environment [Avdeev et al., 2002a,b; Avdeev, 2002]. Among the main features of these codes are the ability to work on PC platforms, and the combination of a fast, but exact state-of-the-art integral equation (IE) approach [Singer, 1995; Pankratov et al., 1995,1997; Singer and Fainberg, 1995,1997] with a modern Krylov subspace iteration [Greenbaum, 1997]. The codes have been validated against both semi-analytical solutions [Chew, 1984; Liu, 1993], 3D IE solutions [Wannamaker, 1991], and 3D FD solutions [Mackie et al., 1993; Alumbaugh et al., 1996; Newman & Alumbaugh, 2002]. Moreover, this series of codes are able to give accurate results for lateral contrast of electrical resistivity as high as 100,000, simulate the responses from DC up to 50 MHz frequency, account for the IP and displacement currents, incoporate anisotropic electrical resistivities, and run large-scale problems involving up to 8,000,000 cells.

In this paper we first review our IE approach following [Avdeev et al., 2002a]. Then we present model examples for induction logging and airborne EM.

2. Theory.

Total $(\mathbf{E},\mathbf{H})$, reference $(\mathbf{E}^{o},\mathbf{H}^{o})$ and scattered $(\mathbf{E}^{S}=\mathbf{E}-\mathbf{E}^{o},\ \mathbf{H}^{S}=\mathbf{H}-\mathbf{H}^{o})$ geoEM fields in 3D earth's models satisfy respective Maxwell's equations:

$$\nabla\times\mathbf{H}=\underline{\underline{\zeta}}(x,y,z,\omega)\mathbf{E}+\mathbf{j}^{ext},\qquad \nabla\times\mathbf{E}=i\omega\underline{\underline{\mu}}(z)(\mathbf{H}+\mathbf{h}^{ext}),\tag{1}$$

$$\nabla\times\mathbf{H}^{o}=\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{o}+\mathbf{j}^{ext},\qquad \nabla\times\mathbf{E}^{o}=i\omega\underline{\underline{\mu}}(z)(\mathbf{H}+\mathbf{h}^{ext},\tag{2}$$

$$\nabla\times\mathbf{H}^{S}=\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{S}+\mathbf{j}^{q},\qquad \nabla\times\mathbf{E}^{S}=i\omega\underline{\underline{\mu}}(z)\mathbf{H}^{S},\tag{3}$$

where
$$\mathbf{j}^{q}=(\underline{\underline{\zeta}}-\underline{\underline{\zeta}}_{o})(\mathbf{E}^{S}+\mathbf{E}^{o}),\tag{4}$$

$\underline{\underline{\zeta}}(x,y,z,\omega)=\underline{\underline{\sigma}}-i\omega\underline{\underline{\varepsilon}}=diag(\zeta_{xx},\zeta_{yy},\zeta_{zz})$ is the diagonal 3×3 matrix of generalized conductivity, $\underline{\underline{\zeta}}_{0}(z,\omega)=diag(\zeta_{0\tau},\zeta_{0\tau},\zeta_{0z})$ is the matrix of the reference conductivity, and $\underline{\underline{\mu}}(z,\omega)=diag(\mu_{\tau},\mu_{\tau},\mu_{z})$. Note that we deliberately chose a 1D formation as the reference one so that we can easily solve Maxwell's equations of (2). Thus hereinafter we assume that the reference fields $(\mathbf{E}^{o},\mathbf{H}^{o})$ are known. From eq.(3) it easy to find that

$$\nabla\times\frac{1}{i\omega}\underline{\underline{\mu}}^{-1}(z)\nabla\times\mathbf{E}^{S}-\underline{\underline{\zeta}}_{o}(z,\omega)\mathbf{E}^{S}=\mathbf{j}^{q}.\tag{5}$$

Applying the Green's function technique to eq. (5), one can derive conventional scattering equation with respect to electric field, cf. [Dmitriev, 1969; Weidelt, 1975]

$$\mathbf{E}^{\mathrm{S}}(\mathbf{r}) = \mathbf{E}_o + Q\mathbf{E}^{\mathrm{S}} = \mathbf{E}_o(\mathbf{r}) + \int_{V^S} \underline{\underline{G}}_o^{ee}(\mathbf{r},\mathbf{r}')(\underline{\underline{\zeta}}(\mathbf{r}') - \underline{\underline{\zeta}}_{\mathrm{o}}(z'))\mathbf{E}^{\mathrm{S}}(\mathbf{r}')dv', \tag{6}$$

where $\underline{\underline{G}}_o^{ee} = \begin{pmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{pmatrix}$ is 3x3 dyadic for electric-to-electric Green's function of 1D reference formation of conductivity $\underline{\underline{\zeta}}_o(z,\omega)$ [Avdeev et al., 1997], $\mathbf{r} = (x,y,z)$, $dv' = dx'\,dy'\,dz'$. Formal solution of eq. (6) is expressed as an infinite Neumann series

$$\mathbf{E}^{\mathrm{S}}(\mathbf{r}) = (1-Q)^{-1}\mathbf{E}_o = \mathbf{E}_o + Q\mathbf{E}_o + Q^2\mathbf{E}_o + \ldots \quad . \tag{7}$$

As a rule, the series given in eq. (7) does not converge. But if we add to both sides of eq. (6) term $\frac{1}{2}\underline{\underline{\lambda}}^{-2}(z)(\underline{\underline{\zeta}}(\mathbf{r}) - \underline{\underline{\zeta}}_{\mathrm{o}}(z))\mathbf{E}^{\mathrm{S}}(\mathbf{r})$ (that is equivalent to the shift of spectrum of Q operator) and change unknown ($\mathbf{E}^{\mathrm{S}} \to \chi$) as

$$\chi = \frac{1}{2}\underline{\underline{\lambda}}^{-1}((\underline{\underline{\zeta}} + \underline{\underline{\zeta}}_{\mathrm{o}}^{*})\mathbf{E}^{\mathrm{S}} + (\underline{\underline{\zeta}} - \underline{\underline{\zeta}}_{\mathrm{o}})\mathbf{E}^{\mathrm{o}}), \tag{8}$$

where $\underline{\underline{\lambda}}(z,\omega) = diag(\sqrt{\operatorname{Re}\zeta_{o\tau}}, \sqrt{\operatorname{Re}\zeta_{o\tau}}, \sqrt{\operatorname{Re}\zeta_{oz}})$, we readily derive the scattering equation of the iterative dissipative method (MIDM) [Singer, 1995; Pankratov et al., 1995;1997; Singer and Fainberg, 1995,1997]

$$\chi(\mathbf{r}) = \chi_o + M\chi = \chi_o(\mathbf{r}) + \int_{V^S} \underline{\underline{K}}(\mathbf{r},\mathbf{r}')(\underline{\underline{\zeta}}(\mathbf{r}') - \underline{\underline{\zeta}}_{\mathrm{o}}(z'))(\underline{\underline{\zeta}}(\mathbf{r}') + \underline{\underline{\zeta}}_{\mathrm{o}}^{*}(z'))^{-1}\chi(\mathbf{r}')dv', \tag{9}$$

where $\underline{\underline{K}}(\xi,\eta,z,z') = \delta(\xi)\delta(\eta)\delta(z-z')\underline{\underline{1}} + 2\underline{\underline{\lambda}}(z)\underline{\underline{G}}_o^{ee}(\xi,\eta,z,z')\underline{\underline{\lambda}}(z')$, δ is the Dirac's delta-function, $\underline{\underline{1}}$ is the identity operator. A remarkable feature of eq. (9) is that it has a contracting kernel, i.e.

$$\|M\chi\| < \|\chi\|, \quad \forall\chi, \tag{10}$$

where $\|\chi\| = \sqrt{\int_{V^S} |\chi(\mathbf{r})|^2 dv}$. From inequality (10) it follows that Neumann series

$$\chi(\mathbf{r}) = (1-M)^{-1}\chi_o = \chi_o + M\chi_o + M^2\chi_o + \ldots \tag{11}$$

converges for any frequency and for any contrast of conductivity. Summation (11) is similar to iteration

$$\chi^{(n+1)} = \chi_o + M\chi^{(n)},\ (n = 1,\ldots). \tag{12}$$

Based on iteration of (12), a number of numerical implementations have been recently developed [Avdeev et al., 1997,1998,1999; Singer et al., 1999; Zhdanov and Fang, 1997]. However, it is easy to figure out that iteration of (12) is nothing more than *simple iteration* for solving the linear system

$$A\chi = \chi_o, \tag{13}$$

where $A = 1 - M$. (It is noteworthy to note that the operator A being non-Hermitian is still pretty well preconditioned, $\kappa(A) = \|A\|\|A^{-1}\| \le \sqrt{C_l}$ $\quad (e.g.\ C_l = 10^4 \Rightarrow \kappa(A) \le 10^2)$, where C_l is the lateral contrast of conductivity.) In [Avdeev et al, 2002a] it is demonstrated that a Krylov subspace iteration [Greenbaum, 1997], outperforms simple iteration of (12) by order of the magnitude. So, in order to solve system of (13) we apply the generalized biconjugate gradient (GPBiCG) method [Zhang, 1997] rather than simple iteration of (12). To stabilize the erratic convergence behavior of this method we have also incorporated the quasi-minimal residual (QMR) smoothing of [Zhou and Walker, 1994] into the iteration scheme. We stop iteration when $r^{(m)} = \left\| \chi_o - A\chi^{(m)} \right\| \cdot \left\| \chi_o \right\|^{-1} \leq 0.003$. Given χ of eq. (13), from eq. (4) we first calculate $\mathbf{j}^q = 2\underline{\lambda}(\underline{\zeta} + \underline{\zeta}_o^*)^{-1}(\underline{\zeta} - \underline{\zeta}_o)(\chi + \underline{\lambda}\mathbf{E}^o)$ at V^S, and, then, find scattered fields at observe points $\mathbf{r}_\alpha$ as

$$\mathbf{H}^S(\mathbf{r}_\alpha) \equiv \int_{V^S} \underline{G}_o^{he}(\mathbf{r}_\alpha; \mathbf{r}')\mathbf{j}^q(\mathbf{r}')dv', \quad \mathbf{E}^S(\mathbf{r}_\alpha) \equiv \int_{V^S} \underline{G}_o^{ee}(\mathbf{r}_\alpha; \mathbf{r}')\mathbf{j}^q(\mathbf{r}')dv', \qquad (14)$$

where $\underline{G}_o^{he}$ is electric-to-magnetic Green's tensor function of the reference formation [Avdeev et al., 1997]. The total fields are obtained by adding the appropriate reference fields to the scattered fields.

3. Induction logging example.

Fig. 1 presents the induction logs for the 3D model of a 45-degree deviated borehole. Curves are also shown for the case of a vertical borehole. The effect of the deviation is clearly seen. Very good agreement is observed between the solutions (discrepancies are less than few percent). Computational statistics for this simulation are listed in Table 1.

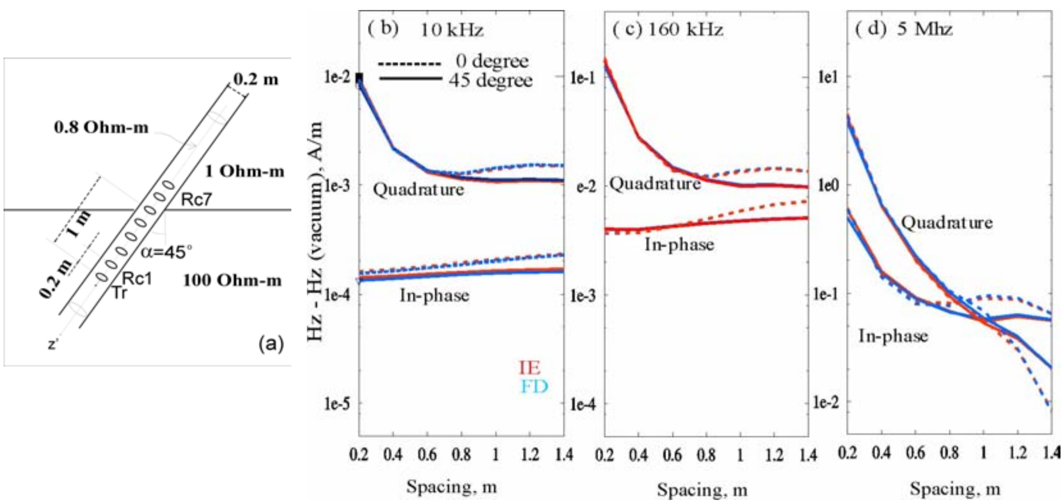

Fig.1 (a) 45-degree deviated borehole intersecting a horizontal boundary; (b-d) comparison of 10-kHz, 160-kHz and 5-MHz responses obtained from the IE (red lines) [Avdeev et al., 2002a] and finite-difference (blue lines) [Newman & Alumbaugh, 2002] solutions (redrawn from [Avdeev et al., 2002a]).

Table 1. The computational statistics (redrawn from [Avdeev et al., 2002a]).

Method	Grid $N_x \times N_y \times N_z$	Frequency, MHz	Iterations-m	Run time[1], c
IE	31×31×32= 30,752	0.01, 0.16, 5	7	2950
FD	563,328 435,334 435,334	0.01, 0.16, 5	1760001200	212156861101

[1]Times are presented for Pentium/350MHz PC (IE code) and for IBM RS-6000 590 workstation (FD code).

4. Airborne EM example.

Fig. 2 shows a model of a vertical fault contact with surface topography. To the right of the fault, the earth has been thrust 10 m upwards. Our IE responses coincide with FD responses of [Alumbaugh et al., 1996] quite well. In order to demonstrate the topography effect the responses without the thrust are also shown.

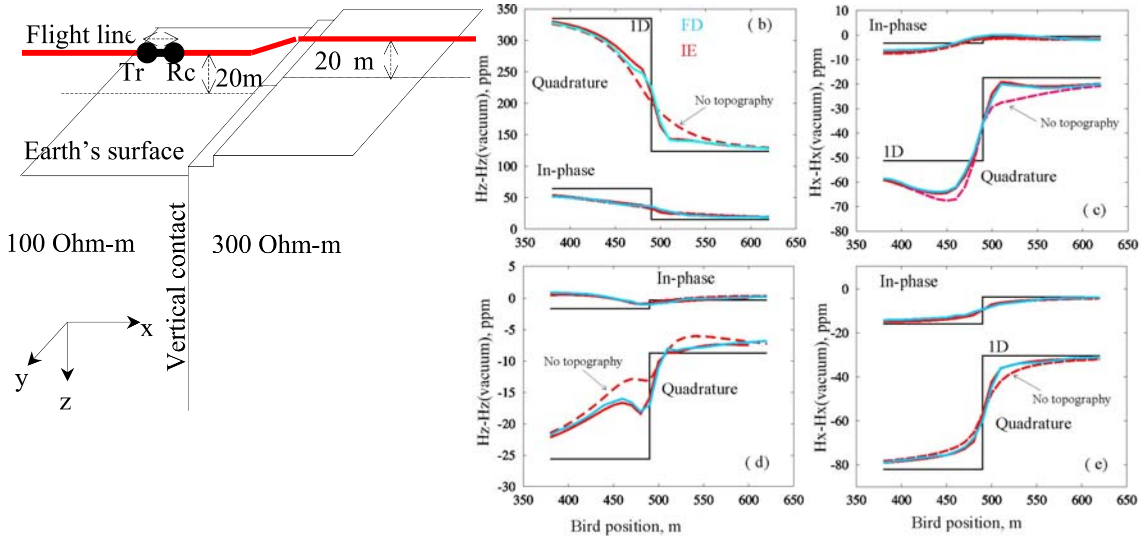

Fig.2. (a) AEM system over a vertical contact and an earth's uplift; 900-Hz AEM responses: (b),(c) VMD exitation; (d),(e) HMD excitation (redrawn from [Avdeev et al.,1998]).

Acknowledgements. The authors thank Gregory A. Newman of the Sandia National Laboratories and Luis S. Martin of the Halliburton Energy Services for fruitful collaboration, support and numerous discussions, which improved the IE solution.

References

[Avdeev, D. B., Kuvshinov, A. V., Pankratov, O. V., and Newman, G.A., 2002a] Three-dimensional induction logging problems, Part I: An integral equation solution and model comparisons: Geophys., 67, 413-426.

[Avdeev, D. B., 2002] Development and application of integral equation method for solving 3D geoelectric problems: Doct. Sci. (habil.) thesis, Geoelectromagnetic Research Institute, Russian Academy of Sciences, Troitsk, 122 pp. (in Russian)

[Avdeev, D. B., Kuvshinov, A. V., Pankratov, O. V., and Martin, S. L., 2002b] Modeling 3D induction logs using non-uniform grids: 16th Workshop on Electromagnetic Induction in the Earth, Santa Fe, USA

[Avdeev, D. B., Kuvshinov, A. V., Pankratov, O. V., and Newman, G. A., 1998] Three-dimensional frequency-domain modelling of airborne electromagnetic responses: Exploration Geophys., 29, 111-119.

[Avdeev, D. B., Kuvshinov, A. V., Pankratov, O. V., & Newman, G.A., 1997] High-performance three-dimensional electromagnetic modeling using modified Neumann series. Wide-band numerical solution and examples: J. Geomagn. Geoelectr., 49, 1519-1539.

[Pankratov, O. V., Kuvshinov, A.V., Avdeev, D. B., 1997] High-performance three-dimensional electromagnetic modeling using modified Neumann series. Anisotropic case: J. Geomagn. Geoelectr., 49, 1541-1547.

[Pankratov, O. V., Avdeev, D. B., Kuvshinov, A. V., 1995] Electromagnetic field scattering in a heterogeneous earth: A solution to the forward problem: Physics of the Solid Earth, 31, 201-209 (English edition).

[Singer, B. Sh., 1995] Method for solution of Maxwell's equations in non-uniform media: Geophys. J. Internat., 120, 590-598.

[Singer, B. Sh. and Fainberg, E.B., 1995] Generalization of the iterative-dissipative method for modeling electromagnetic fields in nonuniform media with displacement currents: J. Appl. Geophys., 34, 41-46.

[Singer, B. Sh. and Fainberg, E.B., 1997] Fast and stable method for 3-D modeling of electromagnetic fields: Exploration Geophys., 28, 130-135.