Synthetic Aperture Imaging through a Dispersive Dielectric Layer

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We consider the problem of synthetic aperture imaging through a known homogeneous layer of (temporally) dispersive material. We seek to make an image of a ¤at earth under the layer from backscattered waves emitted from an antenna following an arbitrary ¤ight path.

We use a linearized scalar model for the wave propagation, namely

$$\nabla^2 E^{sc} - \partial_{tt}(c_0 \varepsilon_r *_t E^{sc}) = -V \partial_{tt} E^{in}, \tag{1}$$

where

$$\varepsilon_r *_t E(t,x) = \int_0^\infty \varepsilon_r(s,x) E(t-s,x) ds, \tag{2}$$

$\epsilon_r(s,x)$ being known and $V(x)$ being unknown. We use any one of a number of formulas for ε_r that may be appropriate (causal) effective-medium models for vegetation. Much of the analysis could also apply to imaging through soil.

Analysis of the problem requires a number of steps:

1. Use an explicit representation for the half-space Green's function in terms of its Fourier transform in time and in the lateral variables.
2. Use 1) to £nd the £eld E^{in} emanating from the antenna. This involves a rudimentary model for the antenna as well as the signal waveform sent to the antenna.
3. Combine 1) and 2) to obtain a mathematical model for the signal received at the antenna.
4. Estimates on ε_r show that the expression obtained in 3) is in the form of a Fourier integral operator F applied to the unknown V.
5. Apply a £ltered backprojection operator Q to the data to make an image. The form of the £lter is obtained from analysis of the composition QF. The theory shows that the resulting image preserves the visible singularities (edges) in the original scene.